Lecture 3
2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

Materials

- https://rf-opto.etti.tuiasi.ro

Materials

- RF-OPTO
  - **https://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **online exam**
  - used at lectures/laboratory

Examen: Logarithmic scales

dB $= 10 \cdot \log_{10} (P_2 / P_1)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

dBm $= 10 \cdot \log_{10} (P / 1\ \text{mW})$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μW
-30 dBm	= 1 μW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

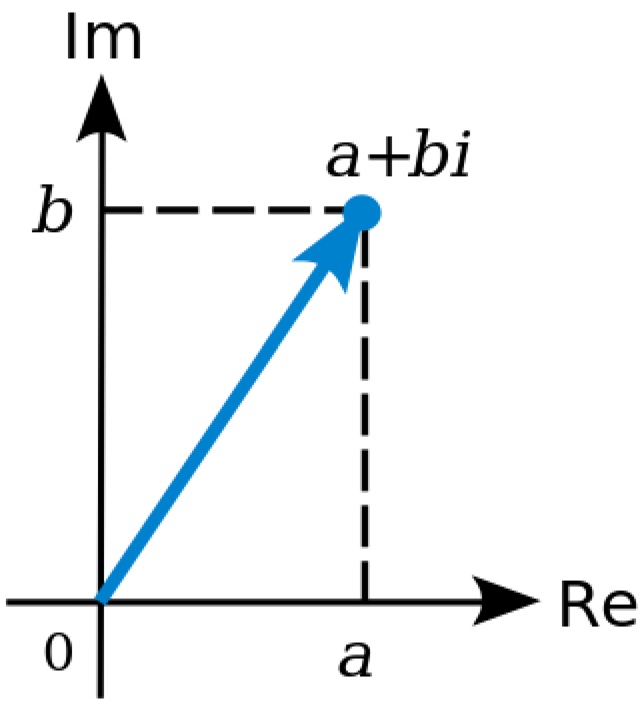
[x] + [dB] = [x]

Exam

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; **$j^2 = -1$**

Complex plane

- abscissa – real part
- ordinate – imaginary part
- any of them can be negative, whole plane, 4 quadrants



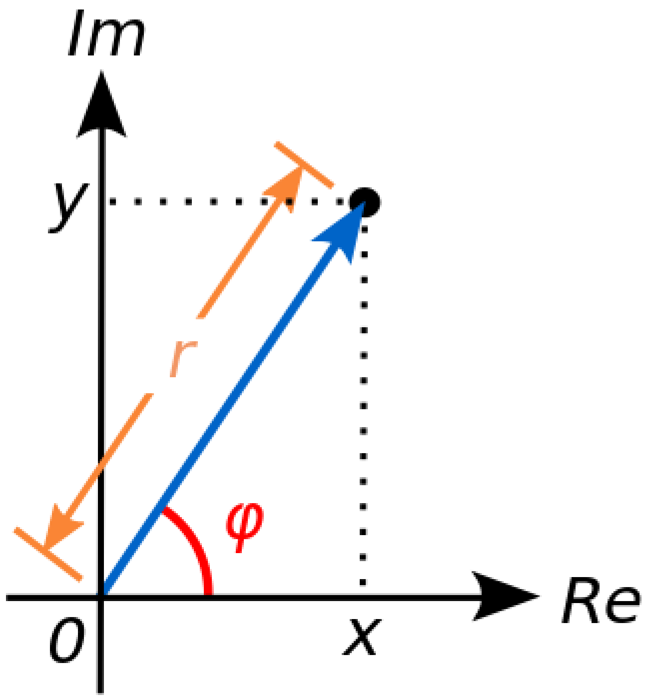
Polar representation

- Polar representation
  - modulus
  - phase relative to the real axis

$$z = a + j \cdot b = |z| \cdot (\cos\varphi + j \cdot \sin\varphi)$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\varphi = \arg(z) = \begin{cases} \arctan\left(\frac{b}{a}\right), & a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi, & a < 0, b \geq 0 \\ \arctan\left(\frac{b}{a}\right) - \pi, & a < 0, b < 0 \\ \frac{\pi}{2}, -\frac{\pi}{2}, nedefinit & a = 0 \end{cases}$$

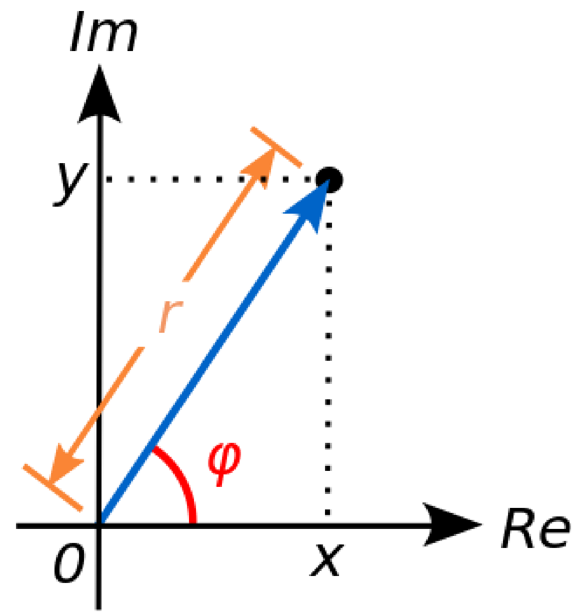


Polar representation

- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot\sin x; \forall x \in R$$

- Polar representation



$$z = a + j\cdot b = |z|\cdot e^{j\cdot\varphi}$$

$$z = a + j\cdot b = |z|\cdot(\cos\varphi + j\cdot\sin\varphi)$$

$$z^n = \left(|z|\cdot e^{j\cdot\varphi}\right)^n = |z|^n\cdot e^{j\cdot n\cdot\varphi} = |z|^n\cdot[\cos(n\cdot\varphi) + j\cdot\sin(n\cdot\varphi)]$$

$$\sqrt{z} = \left(|z|\cdot e^{j\cdot\varphi}\right)^{1/2} = \sqrt{|z|}\cdot e^{j\cdot\frac{\varphi}{2}} = \sqrt{|z|}\cdot\left(\cos\frac{\varphi}{2} + j\cdot\sin\frac{\varphi}{2}\right)$$

$$z\cdot w = |z|\cdot e^{j\cdot\varphi}\cdot|w|\cdot e^{j\cdot\theta} = |z|\cdot|w|\cdot e^{j\cdot(\varphi+\theta)} = |z|\cdot|w|\cdot[\cos(\varphi+\theta) + j\cdot\sin(\varphi+\theta)]$$

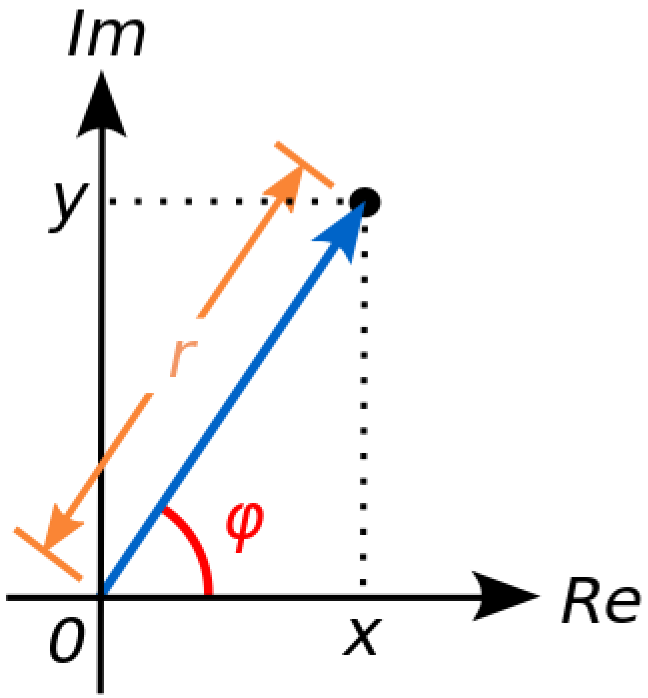
$$z/w = \frac{|z|\cdot e^{j\cdot\varphi}}{|w|\cdot e^{j\cdot\theta}} = \frac{|z|}{|w|}\cdot e^{j\cdot\varphi}\cdot e^{-j\cdot\theta} = \frac{|z|}{|w|}\cdot[\cos(\varphi-\theta) + j\cdot\sin(\varphi-\theta)]$$

Polar representation

- Polar representation

$$|z| = \sqrt{a^2 + b^2}$$

$$|z|^2 = z \cdot z^*$$



$$\left|e^{j\cdot x}\right| = |\cos x + j \cdot \sin x| = \sqrt{\cos^2 x + \sin^2 x} = 1$$

$$\left|e^{j\cdot x}\right| = 1; \quad \forall x \in R$$

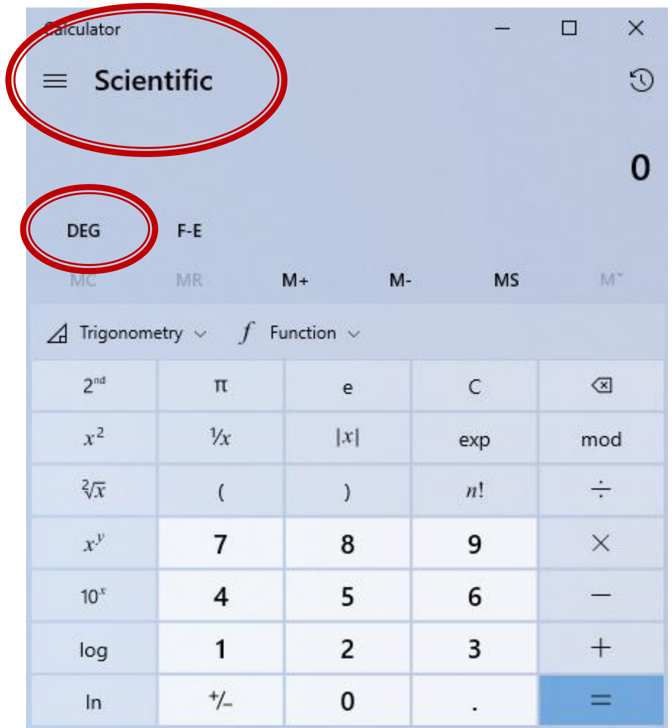
$$z^* = \left(|z| \cdot e^{j\cdot\varphi}\right)^* = |z| \cdot (\cos\varphi + j \cdot \sin\varphi)^* = |z| \cdot (\cos\varphi - j \cdot \sin\varphi) =$$

$$= |z| \cdot [\cos(-\varphi) + j \cdot \sin(-\varphi)] = |z| \cdot e^{-j\cdot\varphi}$$

Polar representation

- standard unit for angles – radians
- microwaves traditional unit for angles – **degrees in decimal form** (55.89°)

$$\varphi = \arg(z) = \begin{cases} \arctan\left(\dfrac{b}{a}\right), & a > 0 \\ \arctan\left(\dfrac{b}{a}\right) + \pi, & a < 0, b \geq 0 \\ \arctan\left(\dfrac{b}{a}\right) - \pi, & a < 0, b < 0 \\ \dfrac{\pi}{2}, -\dfrac{\pi}{2}, nedefinit & a = 0 \end{cases}$$



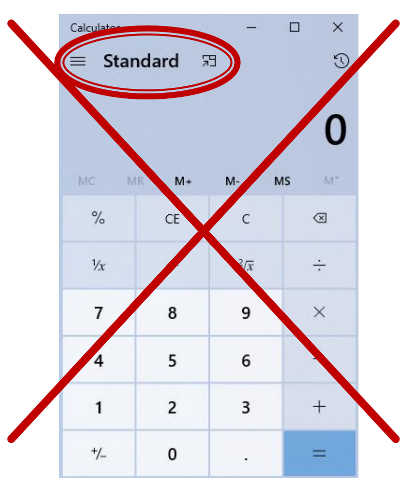
$$\varphi[°] = 180° \cdot \frac{\varphi[rad]}{\pi} \qquad \varphi[rad] = \pi \cdot \frac{\varphi[°]}{180°}$$

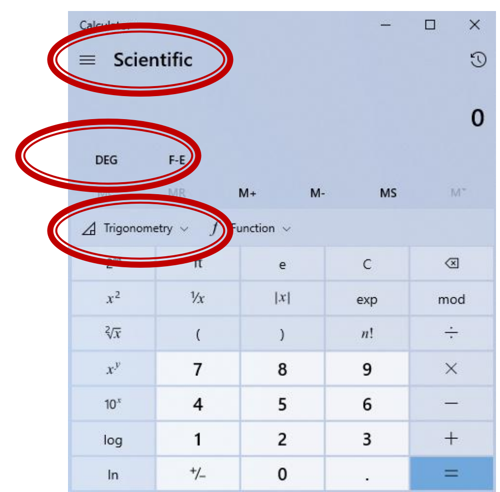
Polar representation

- **Attention to angle numerical values!!**
  - math software – work in standard unit: radians
    - a **conversion** is necessary before and after using a trigonometric function (sin, cos, tan, atan, tanh)
  - scientific calculators have the built-in option of choosing the angle unit
    - always **double check** current working unit

$$\varphi[^\circ] = 180^\circ \cdot \frac{\varphi[rad]}{\pi}$$

$$\varphi[rad] = \pi \cdot \frac{\varphi[^\circ]}{180^\circ}$$





Introduction

~ Microwaves

- Electrical Length (Phase Length)
  - l – physical length
  - E = β·l – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot \left(l \cdot f \cdot \sqrt{\varepsilon_r} \right)$$

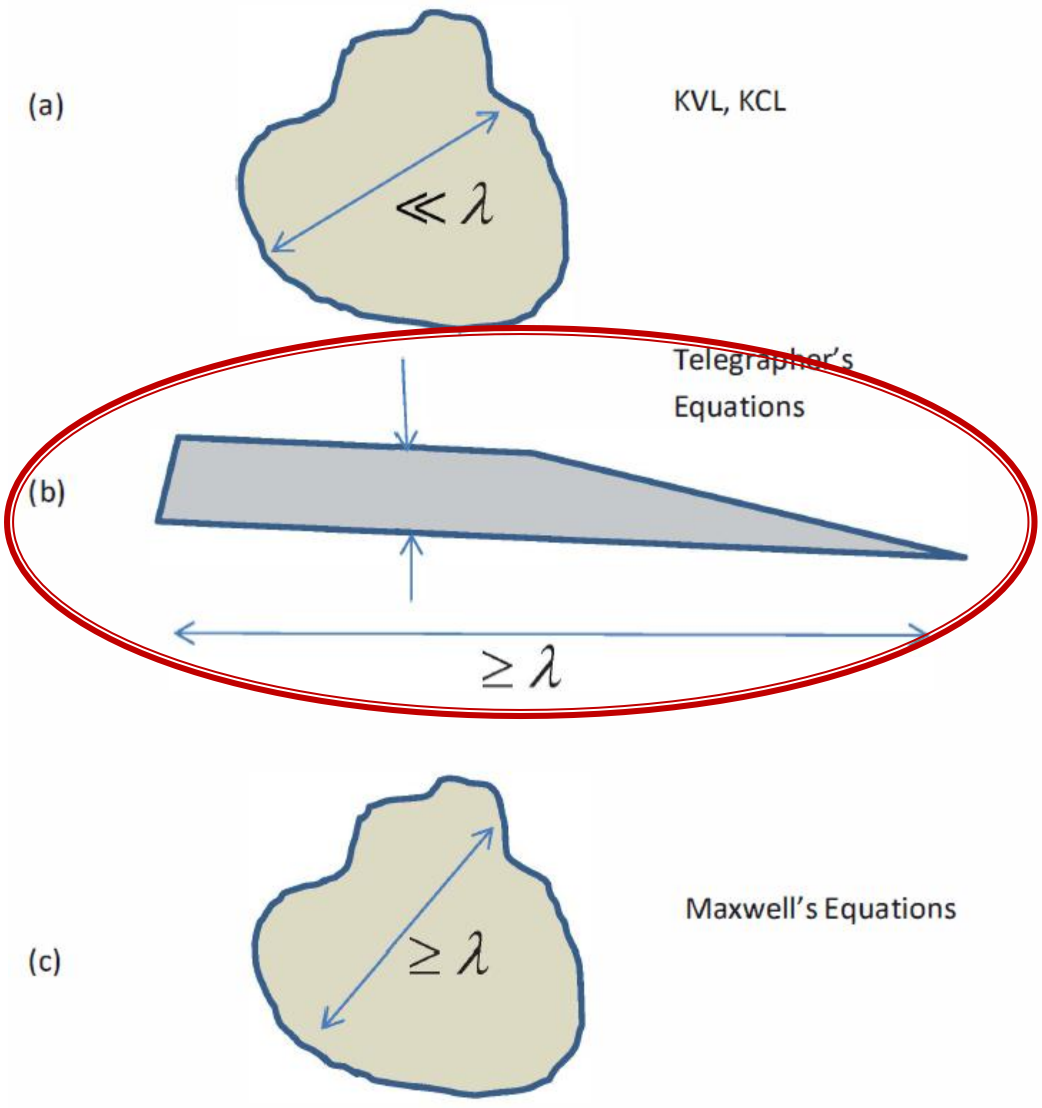
V, I vary
~ useless

- Dependency
  - antenna gain
  - radar cross-section

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



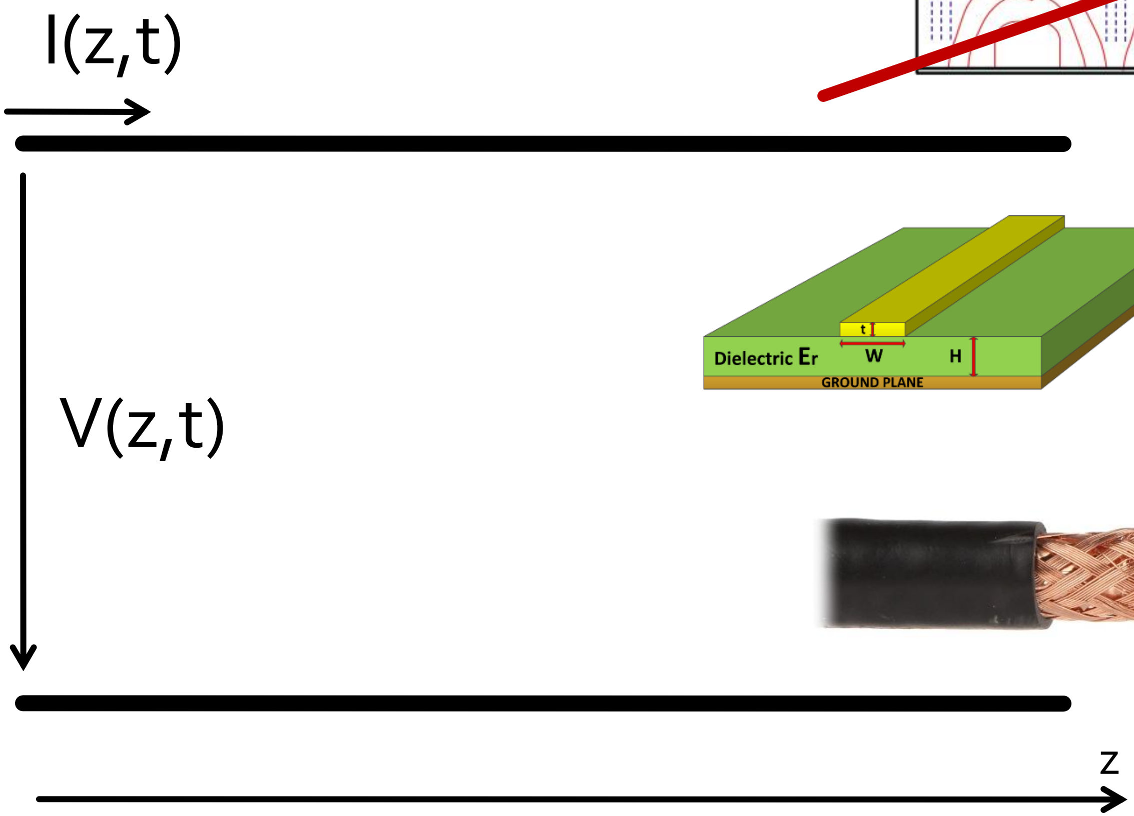
TEM transmission lines

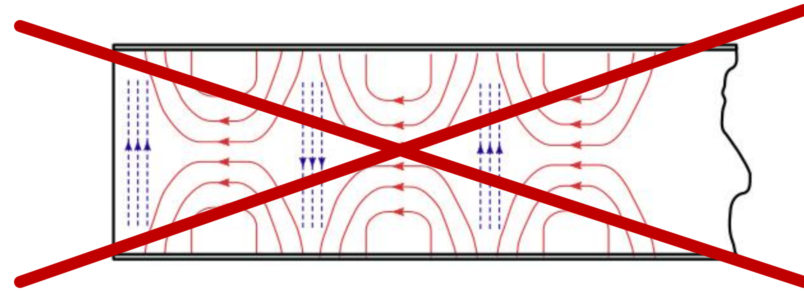
Course Topics

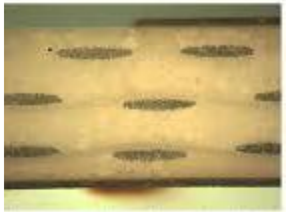
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Transmission line

- TEM wave propagation, at least two conductors

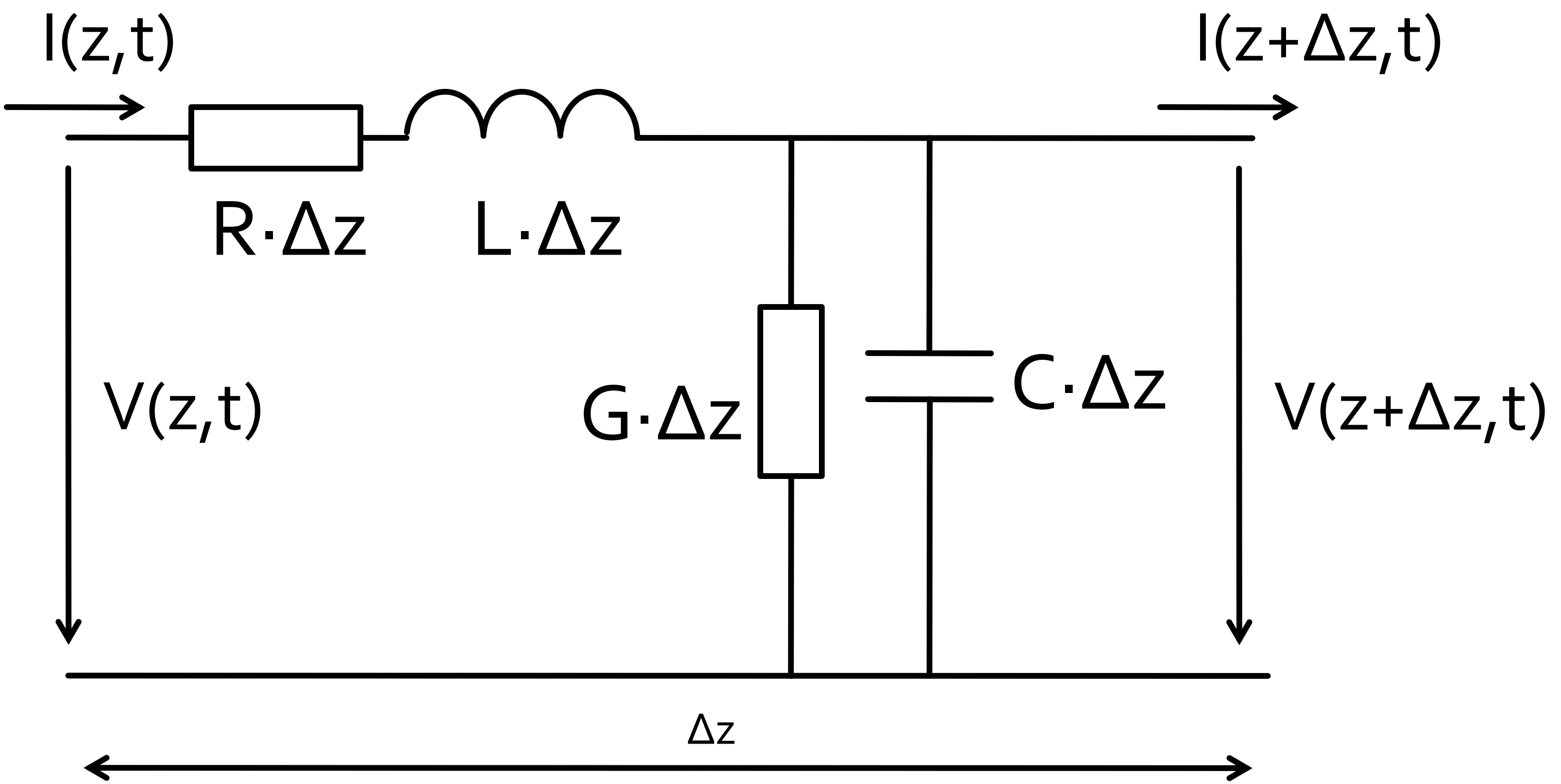






Transmission line equivalent model

- TEM wave propagation, at least two conductors



- **distributed** (line) parameters R, L, G, C (eg. Ω/m)

Telegrapher's equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t}$$ K II

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t}$$ K I

- armonic signals (frequency domain)

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

$$\Big/ \frac{d}{dz}(\ldots)$$

Solving T's E

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \varepsilon\mu + j\omega\mu\sigma$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{cases}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} \left(V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z} \right)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- **Lossless**: R=G=0

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} = j\cdot\omega\cdot\sqrt{L\cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega\cdot\sqrt{L\cdot C}$$

$$Z_0 = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 is **real**

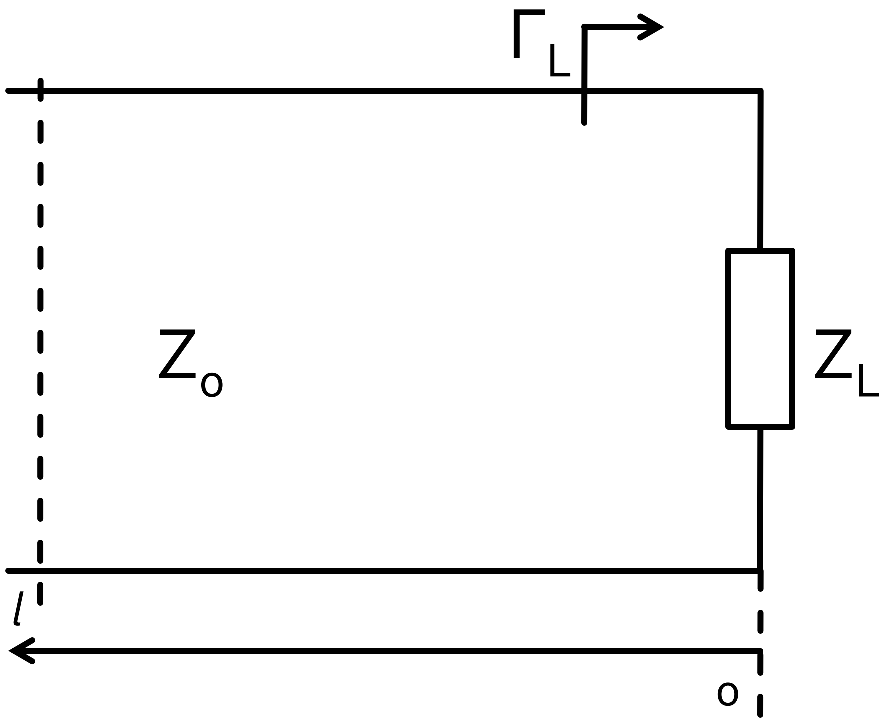
$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$\lambda = \frac{2\pi}{\omega\cdot\sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

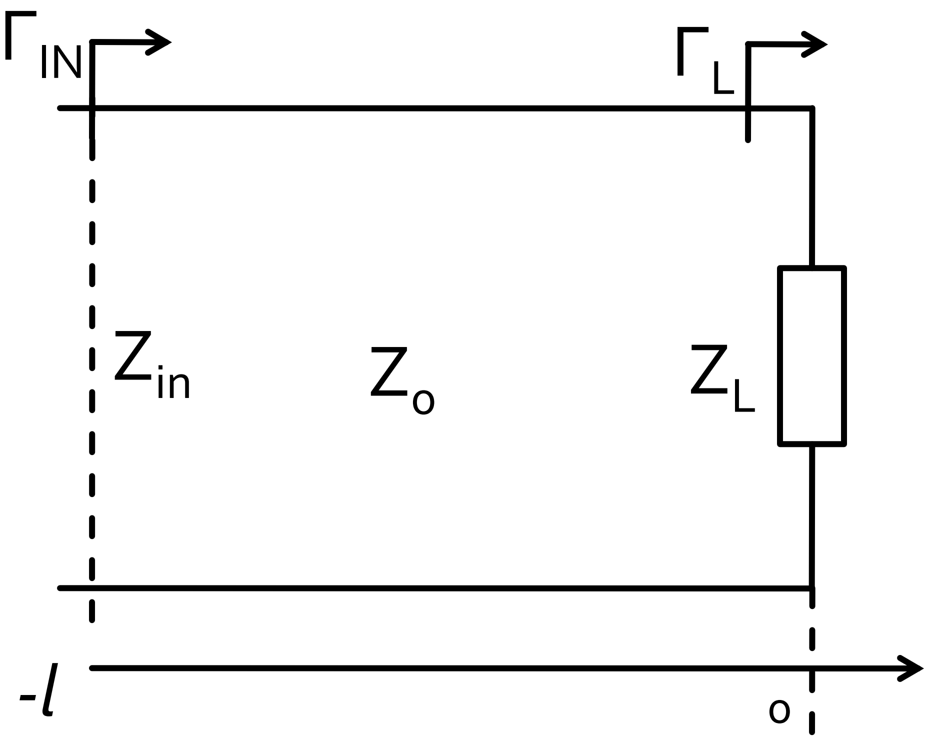
  - Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z)=V_0^+ e^{-j\cdot\beta\cdot z}+V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma=\Gamma(z)=\frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0)=V_0^+ + V_0^-$$

$$\Gamma(0)=\Gamma_L=\frac{V_0^-}{V_0^+}$$

$$V(-l)=V_0^+ e^{j\cdot\beta\cdot l}+V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l)=\Gamma_{IN}=\frac{V_0^-\cdot e^{-j\cdot\beta\cdot l}}{V_0^+\cdot e^{j\cdot\beta\cdot l}}=\Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)|=|\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right|=|\Gamma(0)|$$

$$\boxed{\Gamma_{IN}=\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}\qquad\boxed{|\Gamma_{IN}|=|\Gamma_L|}$$

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \mathrm{Re}\left\{V(z) \cdot I(z)^*\right\} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \mathrm{Re}\left\{1 - \Gamma^* \cdot e^{-2j\cdot\beta\cdot z} + \Gamma \cdot e^{2j\cdot\beta\cdot z} - |\Gamma|^2\right\}$$

$$\left(z - z^*\right) = \mathrm{Im}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - |\Gamma|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB]

$$\mathrm{RL} = -20 \cdot \log|\Gamma| \quad [\mathrm{dB}]$$

The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

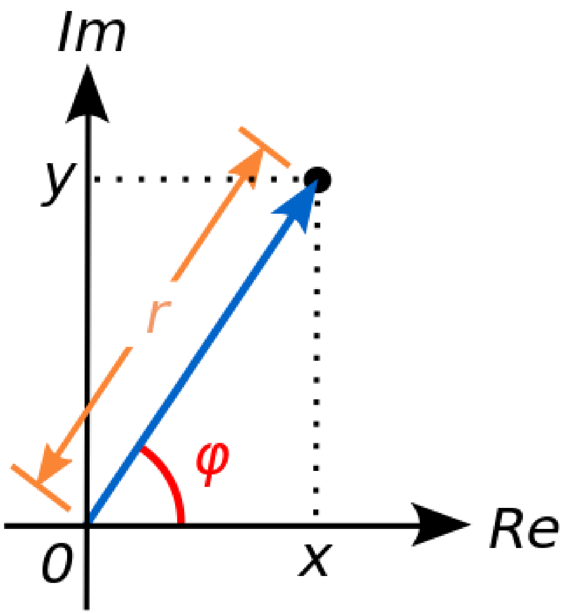
- Average power flow is constant along the line
  - (**no** P_{avg}(**z**))
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very “energetic” (basic physics) point of view
  - more power = “more” signal

Polar representation

- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot \sin x; \forall x \in R$$

- Polar representation



$$z = a + j\cdot b = |z|\cdot e^{j\cdot\varphi}$$

$$z = a + j\cdot b = |z|\cdot(\cos\varphi + j\cdot\sin\varphi)$$

$$z^n = \left(|z|\cdot e^{j\cdot\varphi}\right)^n = |z|^n\cdot e^{j\cdot n\cdot\varphi} = |z|^n\cdot[\cos(n\cdot\varphi) + j\cdot\sin(n\cdot\varphi)]$$

$$\sqrt{z} = \left(|z|\cdot e^{j\cdot\varphi}\right)^{1/2} = \sqrt{|z|}\cdot e^{j\cdot\frac{\varphi}{2}} = \sqrt{|z|}\cdot\left(\cos\frac{\varphi}{2} + j\cdot\sin\frac{\varphi}{2}\right)$$

$$z\cdot w = |z|\cdot e^{j\cdot\varphi}\cdot|w|\cdot e^{j\cdot\theta} = |z|\cdot|w|\cdot e^{j\cdot(\varphi+\theta)} = |z|\cdot|w|\cdot[\cos(\varphi+\theta) + j\cdot\sin(\varphi+\theta)]$$

$$z/w = \frac{|z|\cdot e^{j\cdot\varphi}}{|w|\cdot e^{j\cdot\theta}} = \frac{|z|}{|w|}\cdot e^{j\cdot\varphi}\cdot e^{-j\cdot\theta} = \frac{|z|}{|w|}\cdot[\cos(\varphi-\theta) + j\cdot\sin(\varphi-\theta)]$$

Polar representation

- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot\sin x; \forall x \in R$$

$$e^{j\cdot x} + e^{-j\cdot x} = \cos x + j\cdot\sin x + \cos(-x) + j\cdot\sin(-x)$$

$$e^{j\cdot x} + e^{-j\cdot x} = \cos x + j\cdot\sin x + \cos x - j\cdot\sin x = 2\cdot\cos x$$

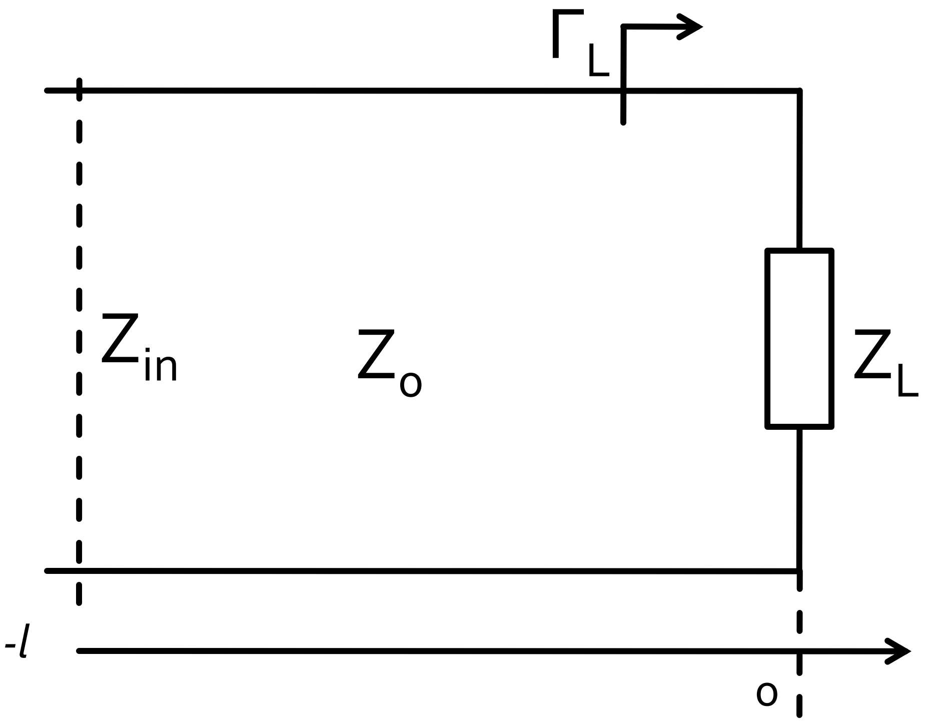
$$\longrightarrow \quad \cos x = \frac{e^{j\cdot x} + e^{-j\cdot x}}{2}$$

$$e^{j\cdot x} - e^{-j\cdot x} = \cos x + j\cdot\sin x - \cos(-x) - j\cdot\sin(-x)$$

$$e^{j\cdot x} - e^{-j\cdot x} = \cos x + j\cdot\sin x - \cos x + j\cdot\sin x = 2j\cdot\sin x$$

$$\longrightarrow \quad \sin x = \frac{e^{j\cdot x} - e^{-j\cdot x}}{2j}$$

The lossless line



$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\cdot\beta\cdot l} - \frac{V_0^-}{Z_0} e^{-j\cdot\beta\cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \qquad Z_{in} = Z_0 \cdot \frac{1 + \Gamma \cdot e^{-2j\cdot\beta\cdot l}}{1 - \Gamma \cdot e^{-2j\cdot\beta\cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} + (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} - (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0 \cdot \tan(\beta\cdot l)}{Z_0 + j\cdot Z_L \cdot \tan(\beta\cdot l)}$$

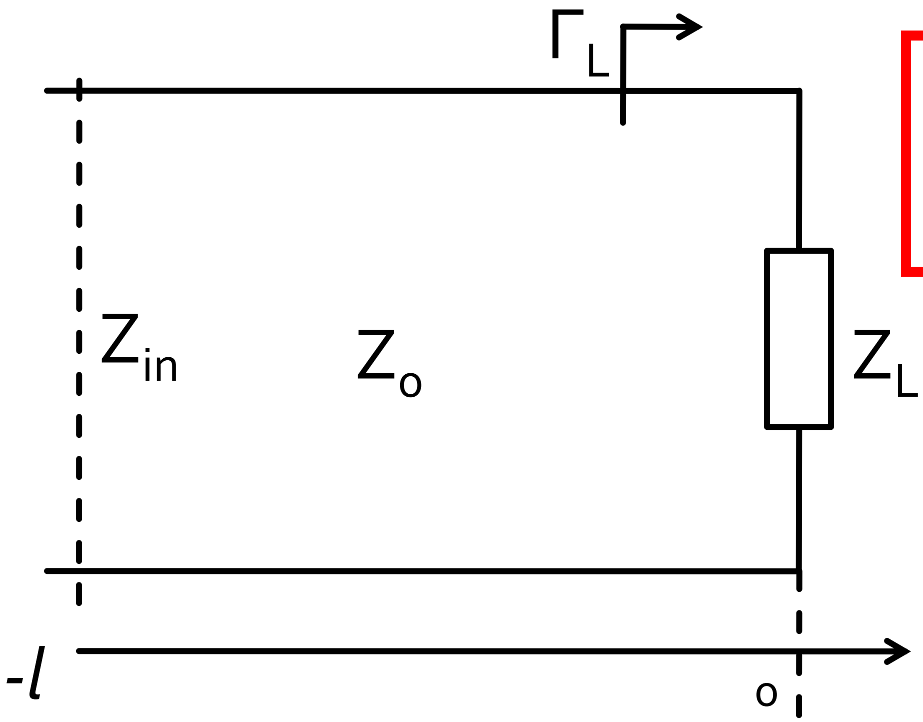
The lossless line

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

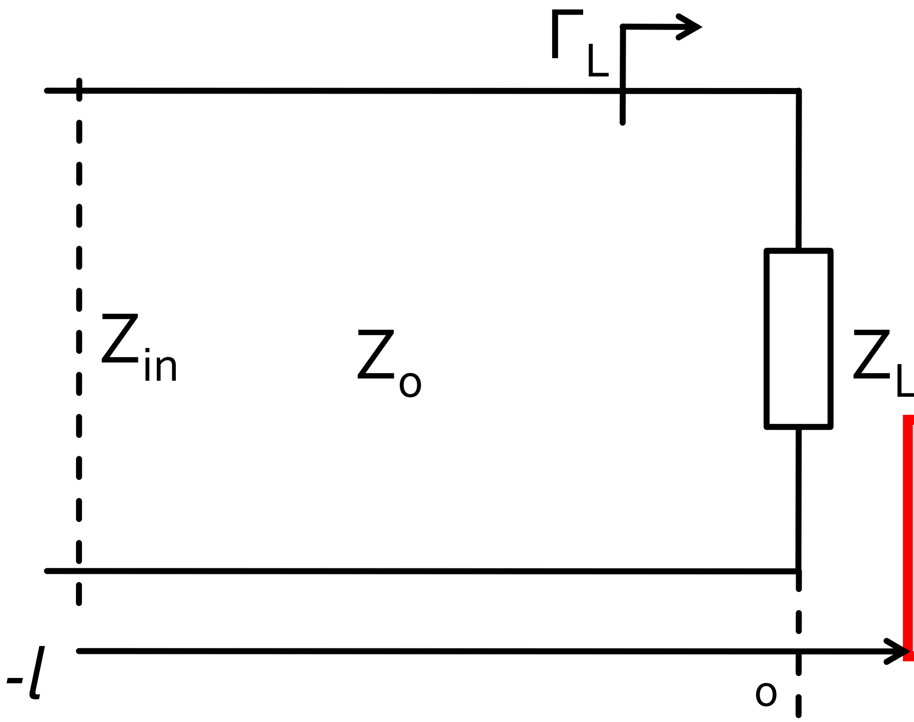
- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

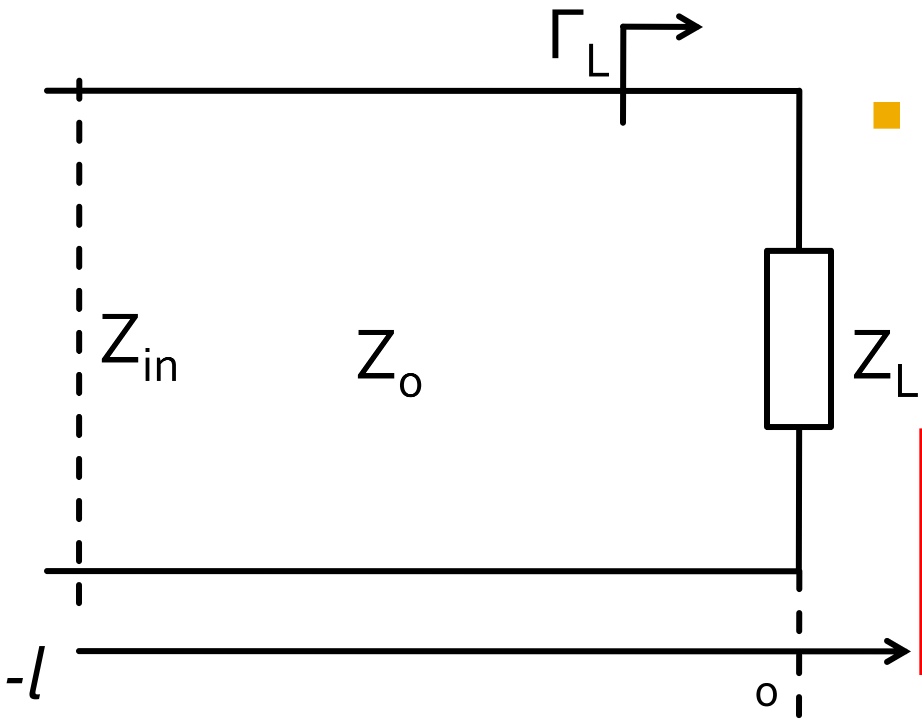
The lossless line, special cases

- $l = k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0$

$$Z_{in} = Z_L$$

- $l = \lambda/4 + k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{\pi}{2} + k \cdot \pi \quad \tan \beta \cdot l \rightarrow \infty$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



- quarter-wave transformer
  - E=90°

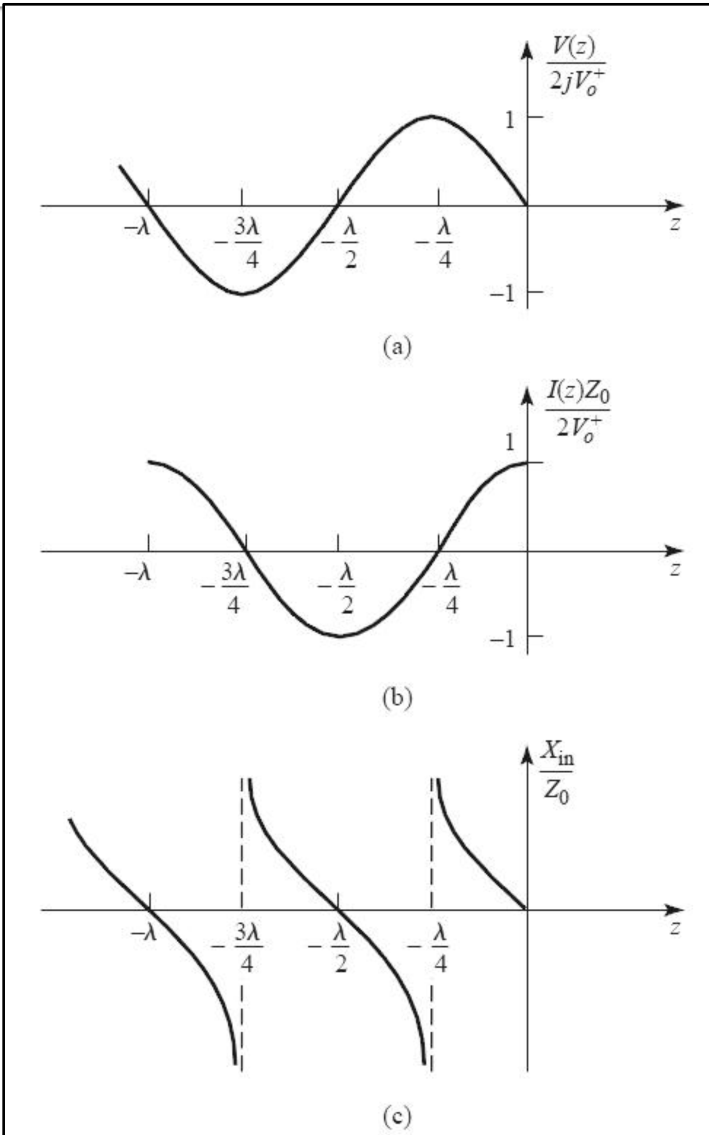
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Short-circuited transmission line

- $Z_L = 0$
- input purely **imaginary** for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

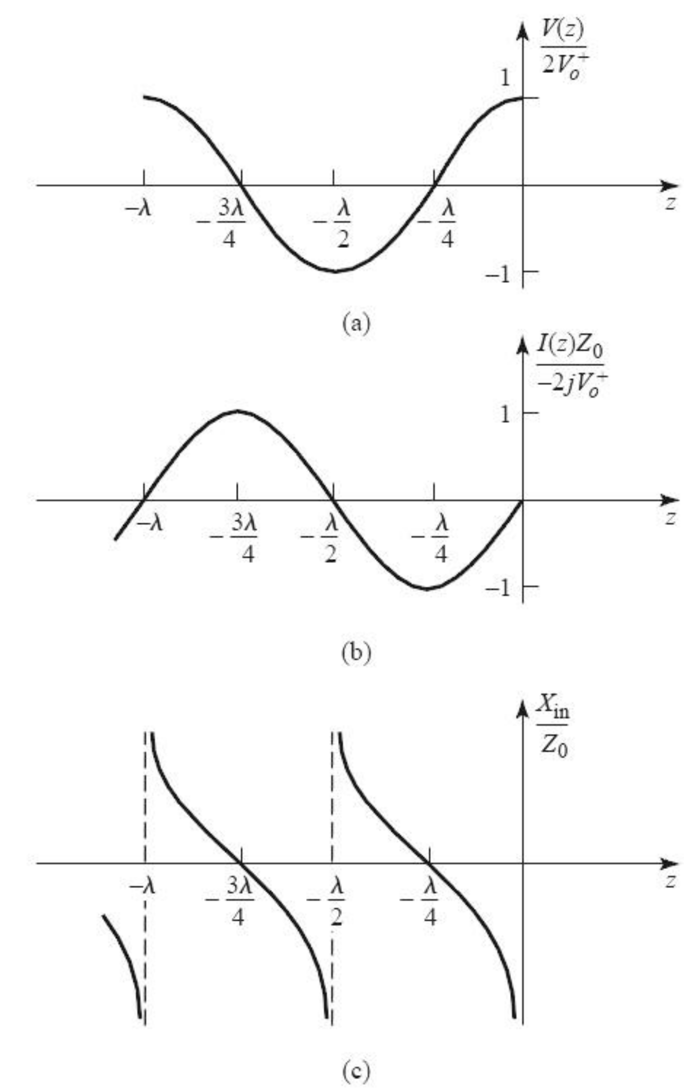


Open-circuited transmission line

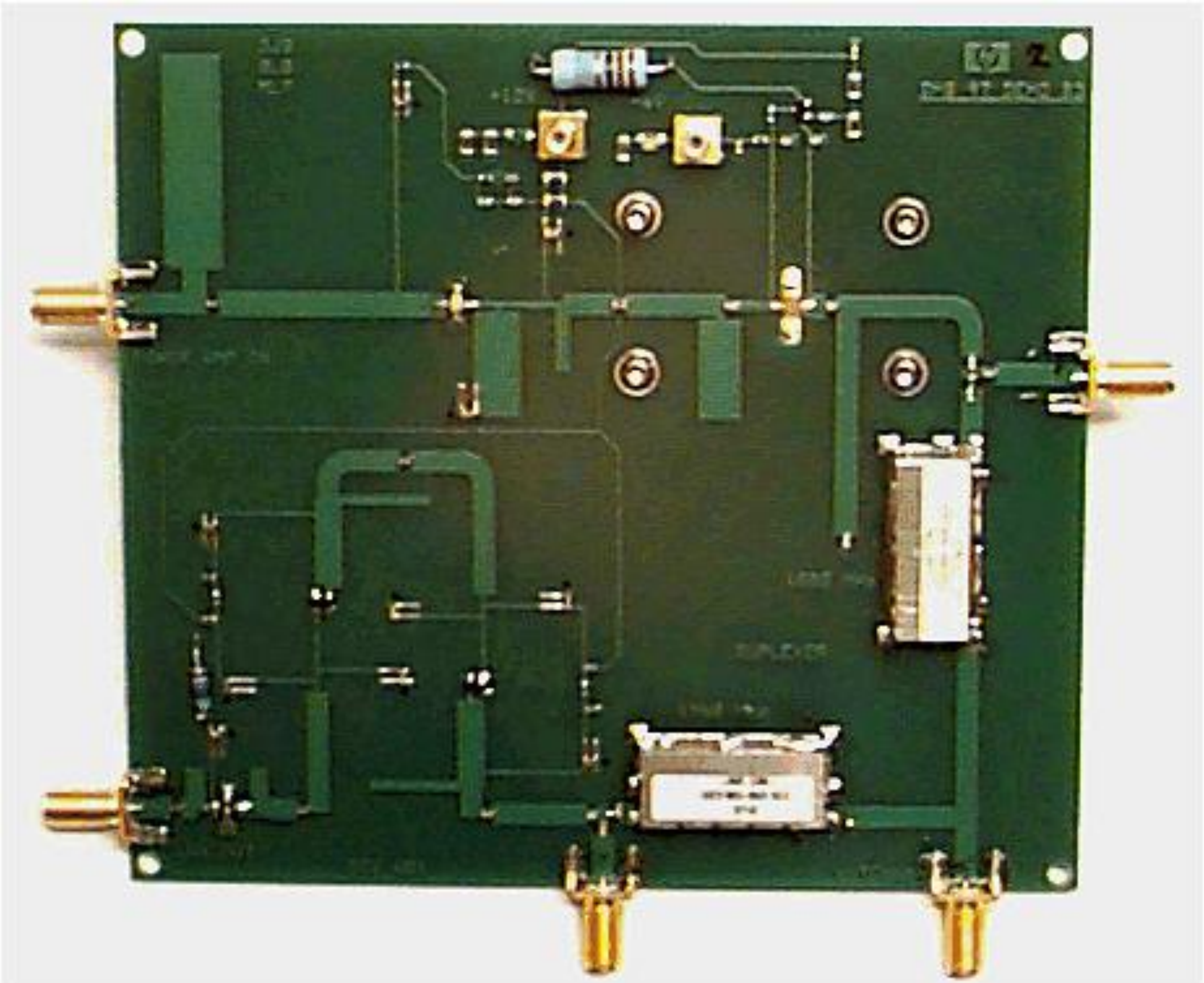
- $Z_L = \infty \longrightarrow 1 / Z_L = 0$
- input purely **imaginary** for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

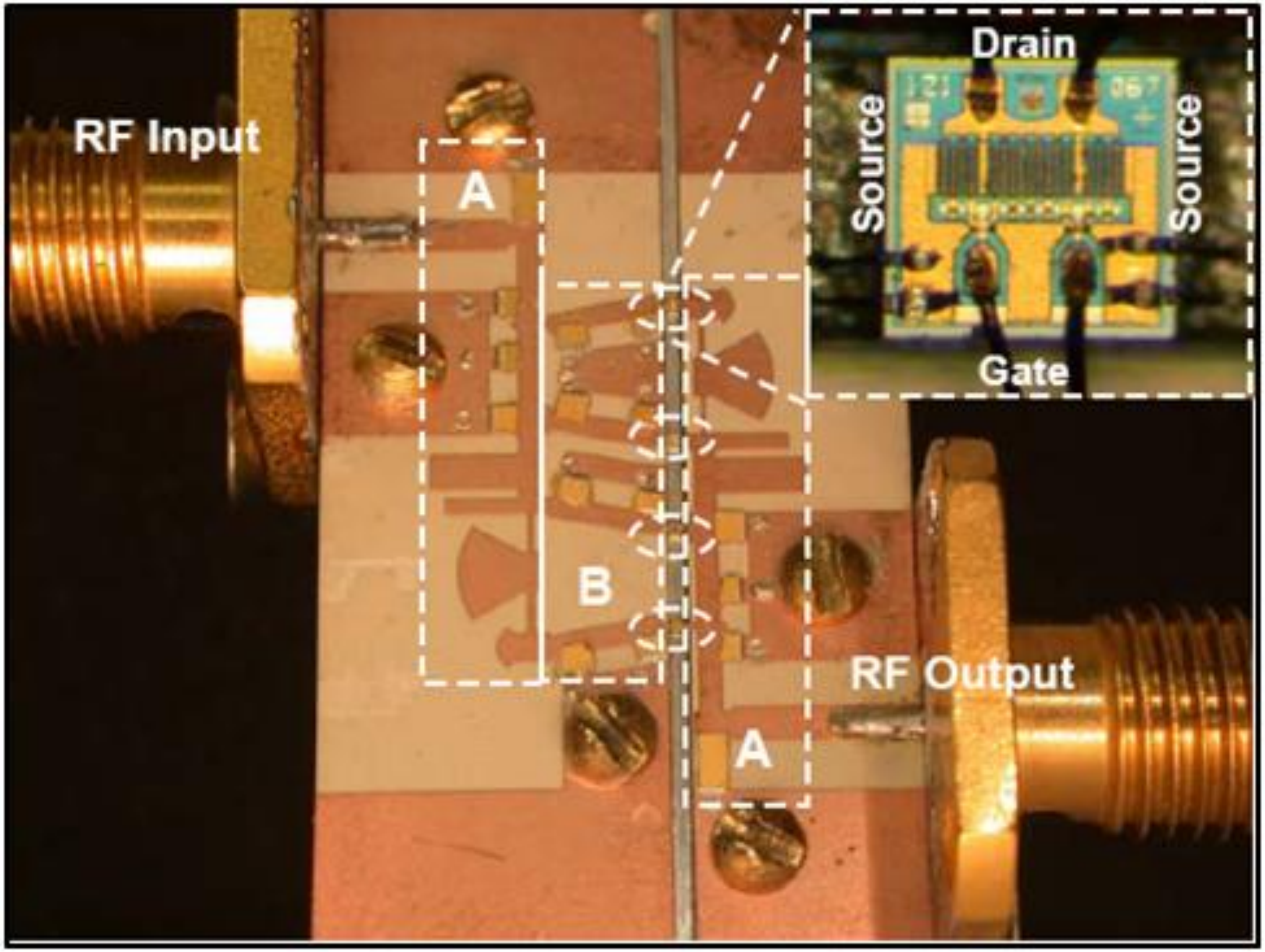
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



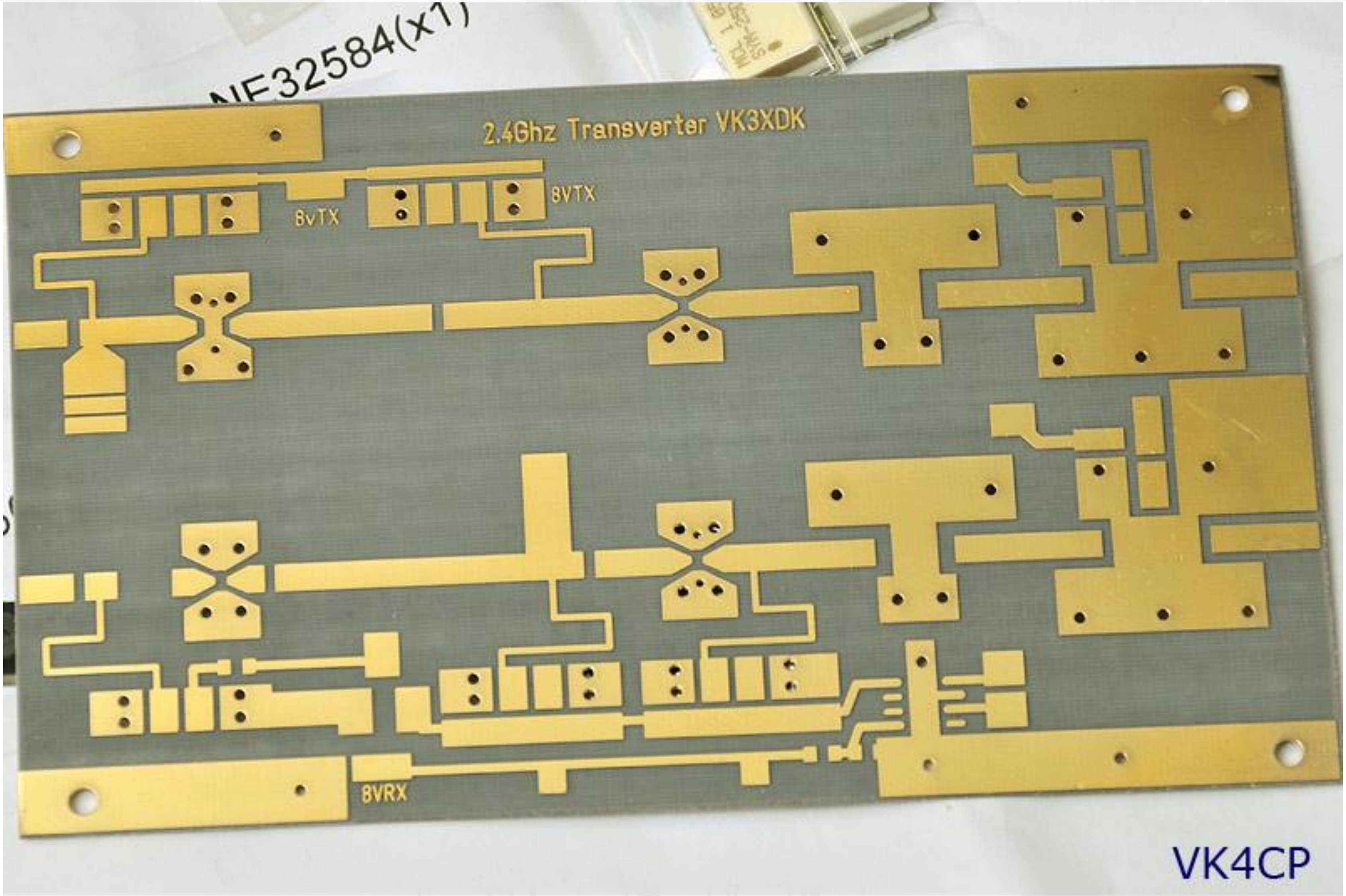
Examples



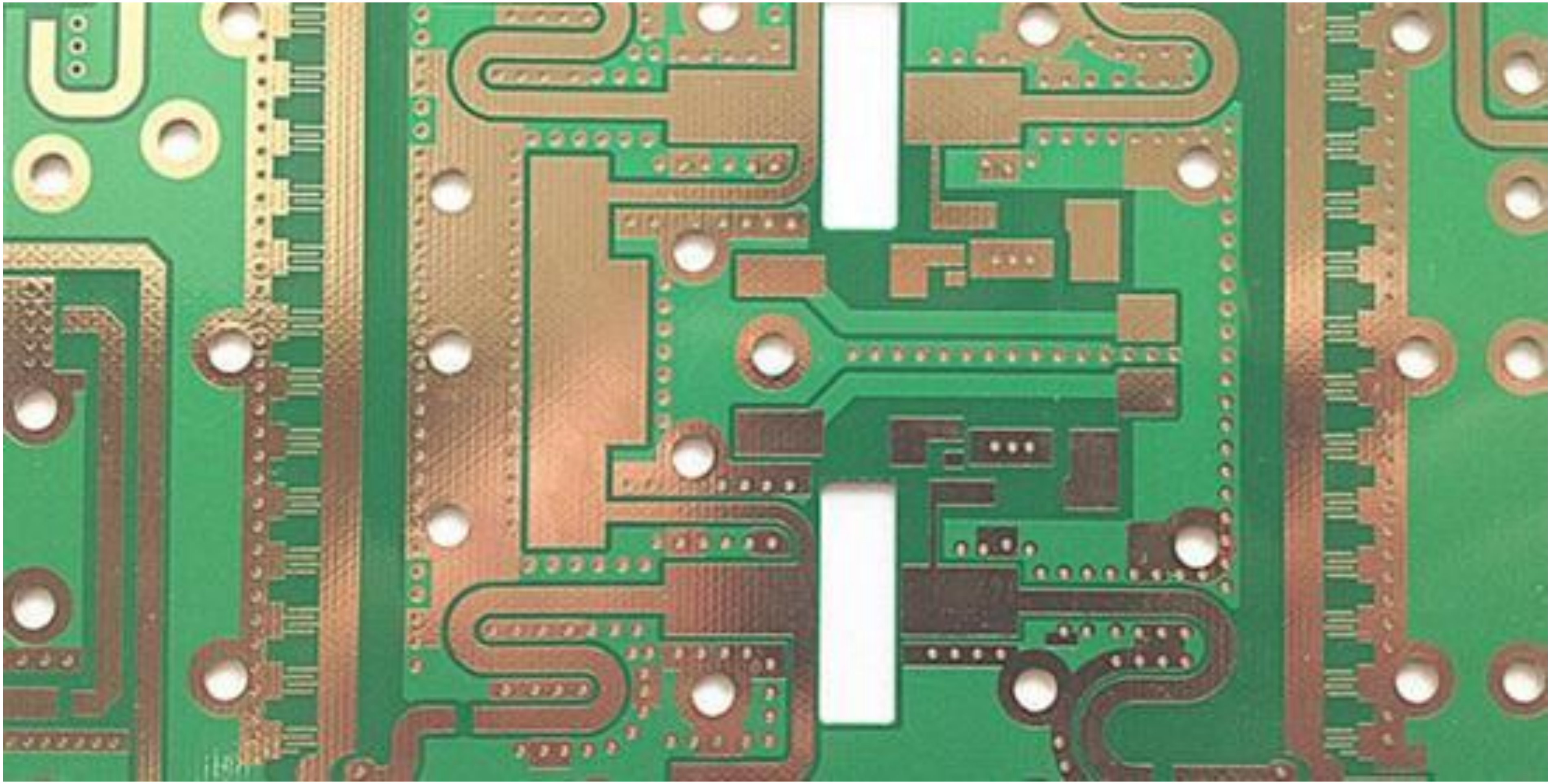
Examples



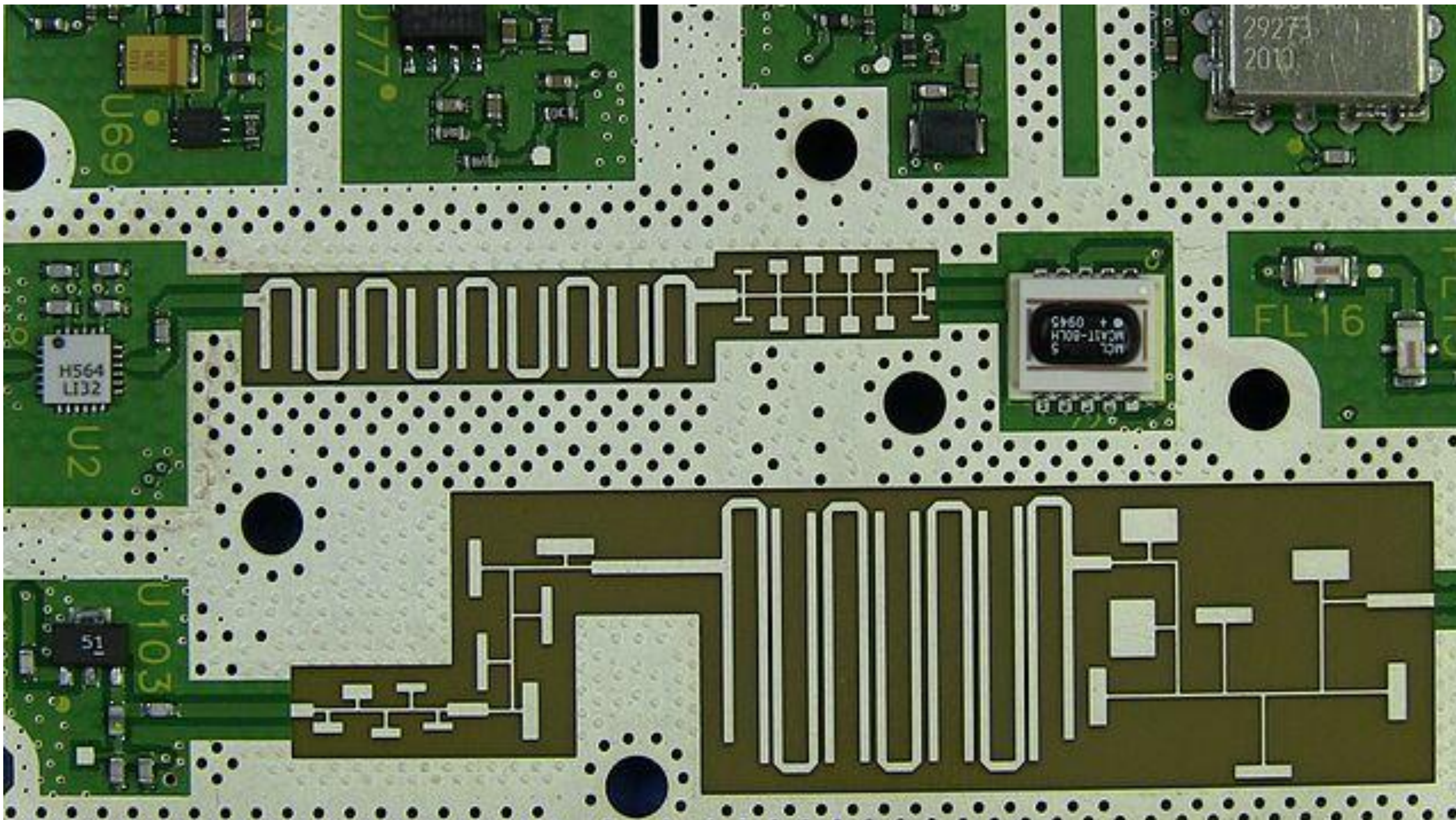
Examples



Examples



Examples



Voltage standing wave ratio

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad |V(z)| = |V_0^+| \cdot |e^{-j\cdot\beta\cdot z}| \cdot |1 + \Gamma \cdot e^{2j\cdot\beta\cdot z}| \qquad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\cdot\beta\cdot z}|$$

maximum magnitude value for $e^{\theta+2j\cdot\beta\cdot z} = 1$ $\qquad V_{\max} = |V_0^+| \cdot (1 + |\Gamma|)$

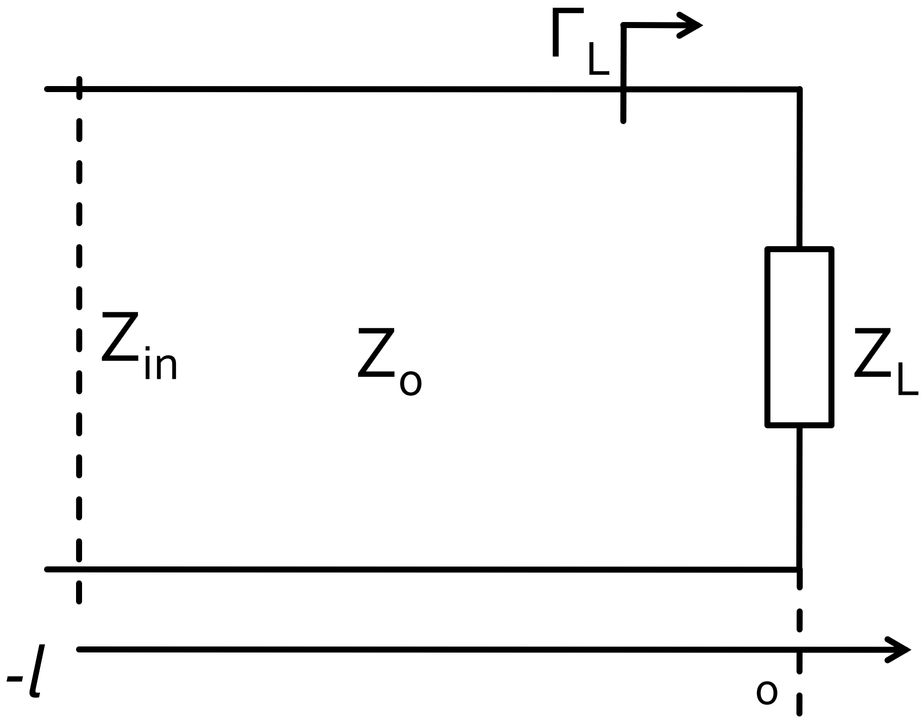
minimum magnitude value for $e^{\theta+2j\cdot\beta\cdot z} = -1$ $\qquad V_{\min} = |V_0^+| \cdot (1 - |\Gamma|)$

- SWR is defined as the ratio between maximum and minimum
  - (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

  - real number $1 \le VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2 j \cdot \beta \cdot l}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2 j \cdot \beta \cdot l}$$

Power transfer

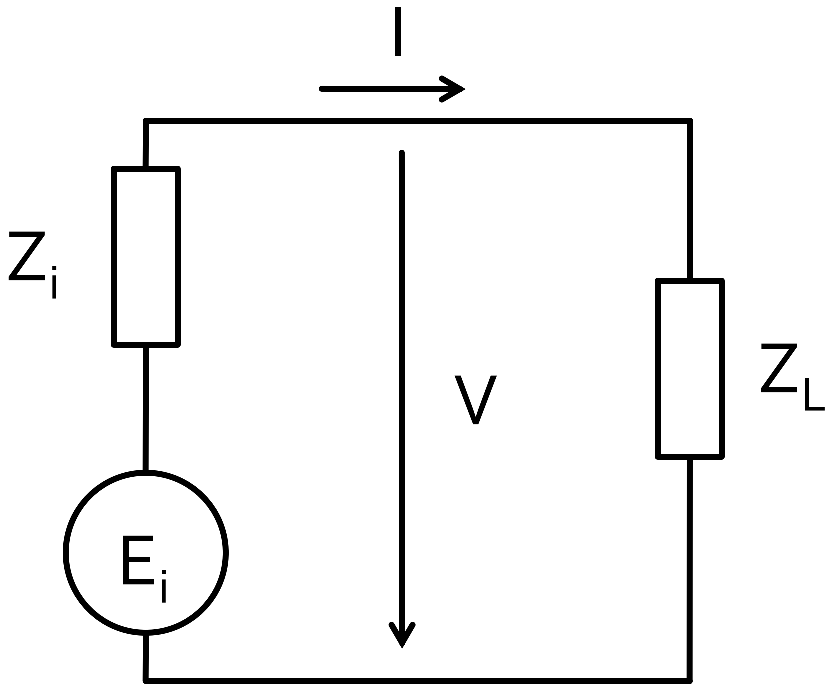
Impedance Matching

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Matching

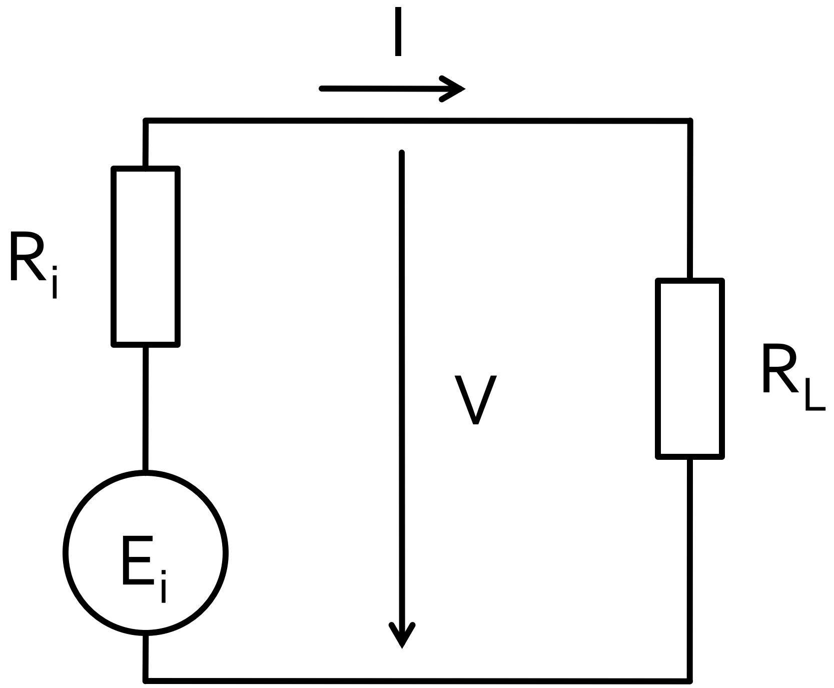
- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

Matching, real impedances

- Source matched to load



$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

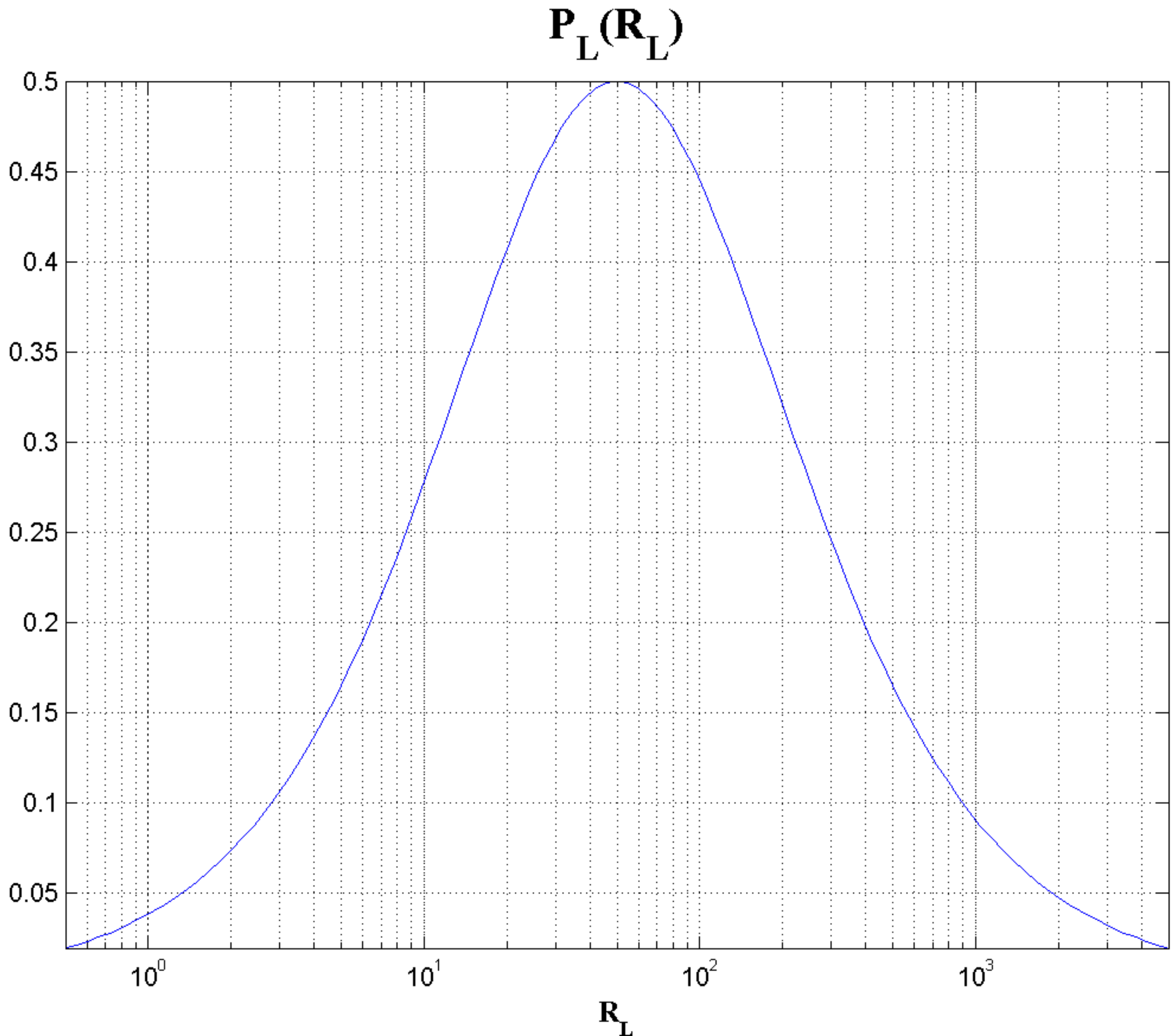
$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

Matching, real impedances

$$P_L = R_L \cdot I^2 \qquad P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

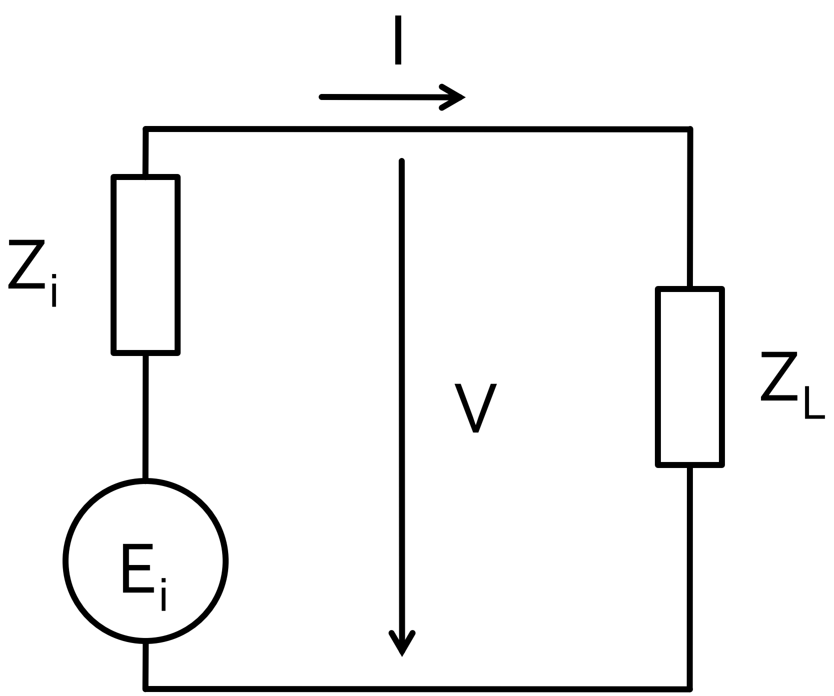
- Power dissipated on load
  - $R_i = 50\Omega$
  - $R_L = 0 \rightarrow P_L = 0$
  - $R_L = \infty \rightarrow P_L = 0$

Matching, real impedances



Matching, complex impedances

- Source matched to load



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \mathrm{Re}\left\{ Z_L \cdot |I|^2 \right\}$$

$$P_L = \mathrm{Re}\{ Z_L \} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Matching

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

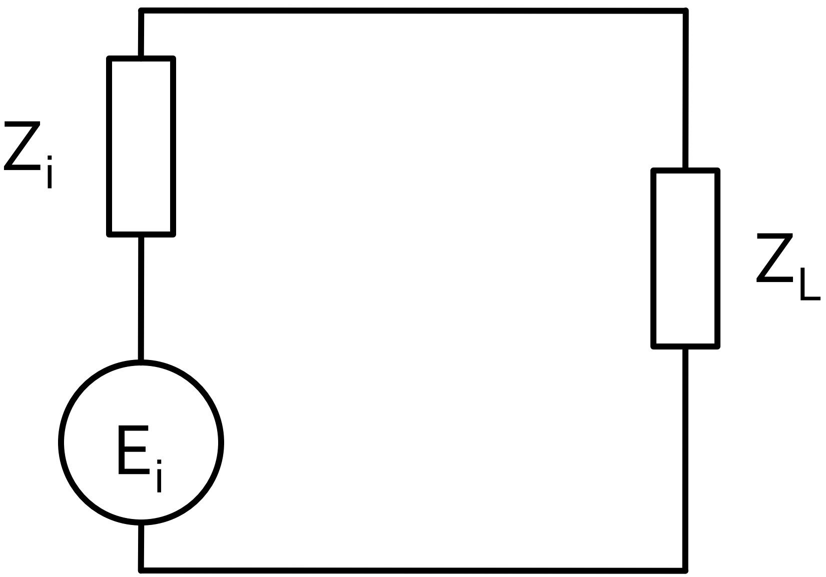
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Matching
  - maximum power transmitted to the load
  - condition?

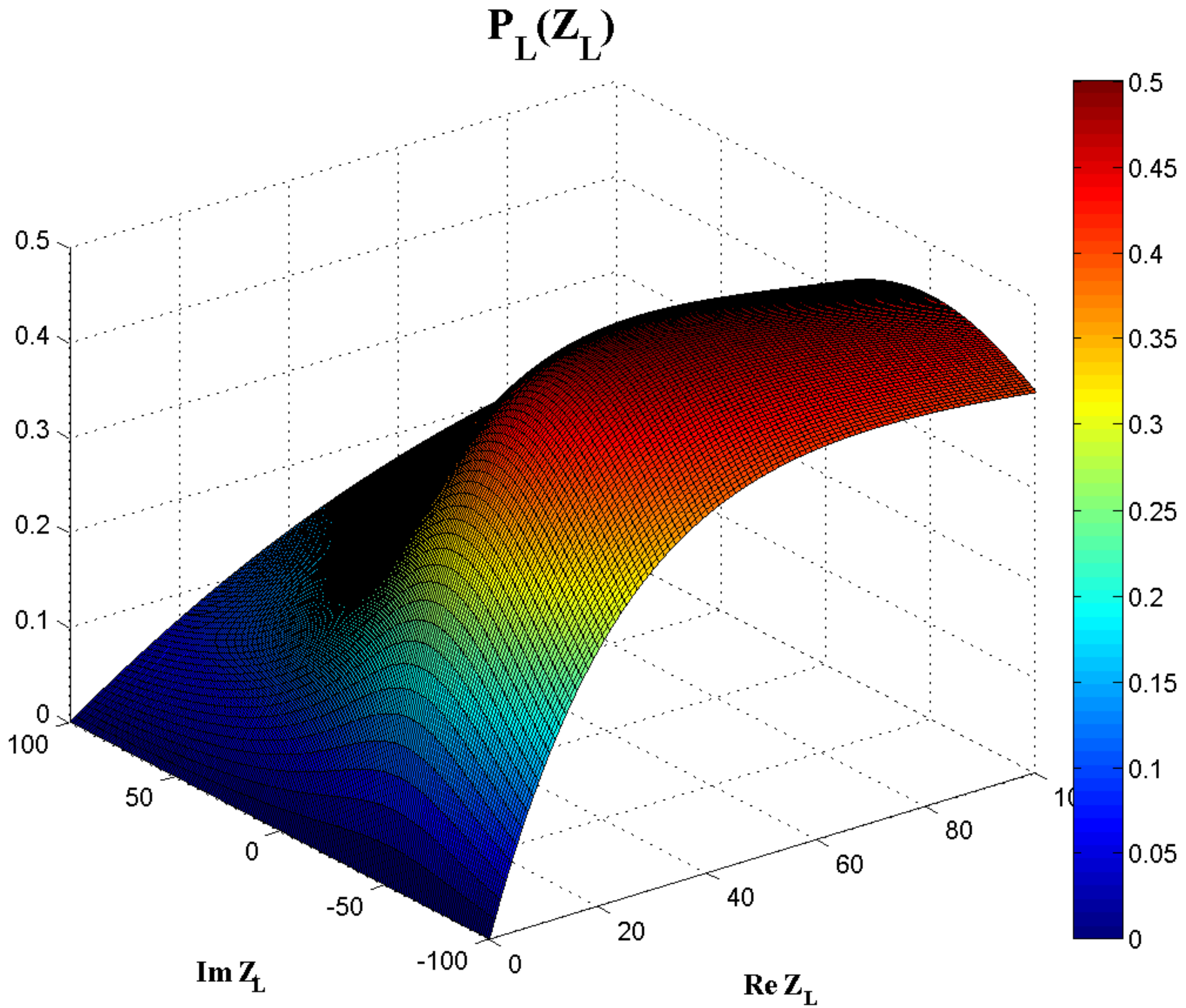
Matching, example

- $E = 10V$
- $Z_i = 50\,\Omega + j \cdot 50\Omega$
- $P_L(Z_L)$ **?**

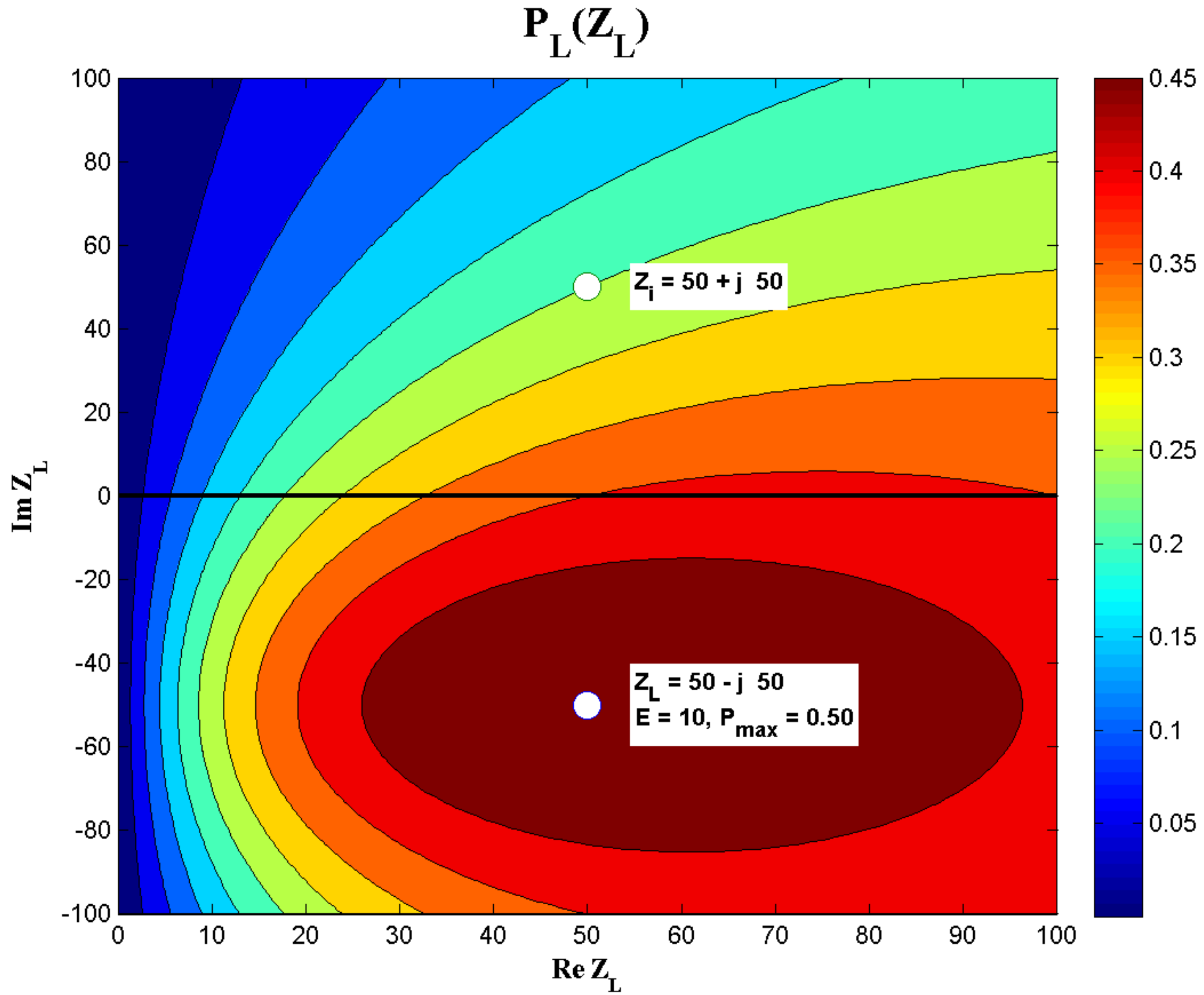
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



Matching, example



Matching, example



Matching , from the point of view of power transmission

$$R_i > 0,\ R_L > 0$$

$$P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a$$

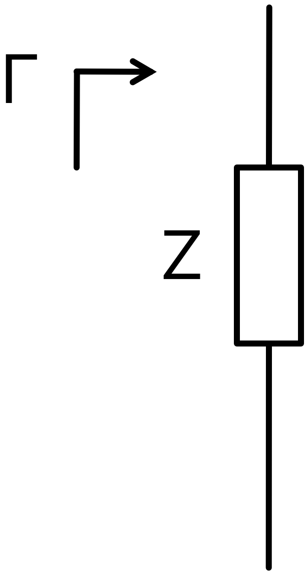
$$R_L = R_i,\ X_L = -X_i$$

- $P_{Lmax} = P_a$: Available Power

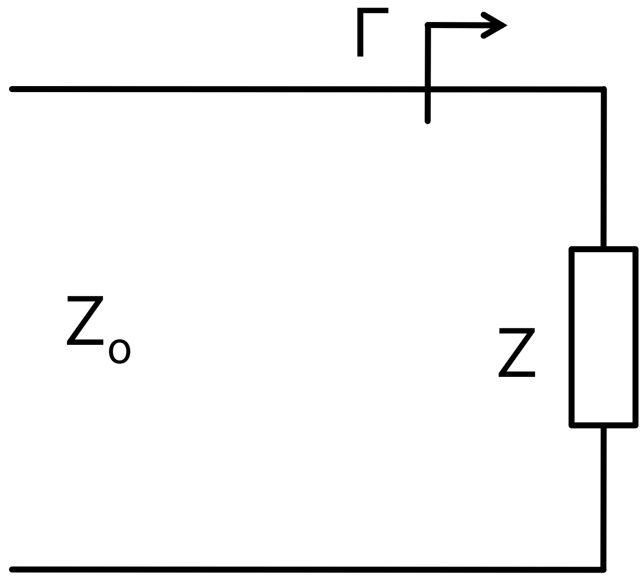
$$Z_L = Z_i^*$$

Reflection coefficient

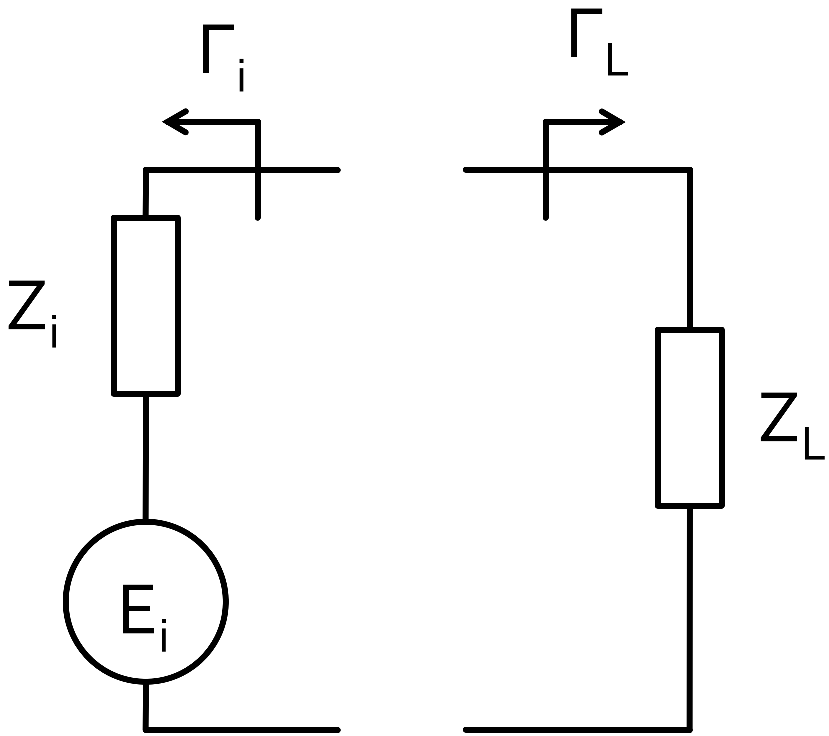
- Any impedance Z_o chosen as reference



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



Matching , from the point of view of power transmission



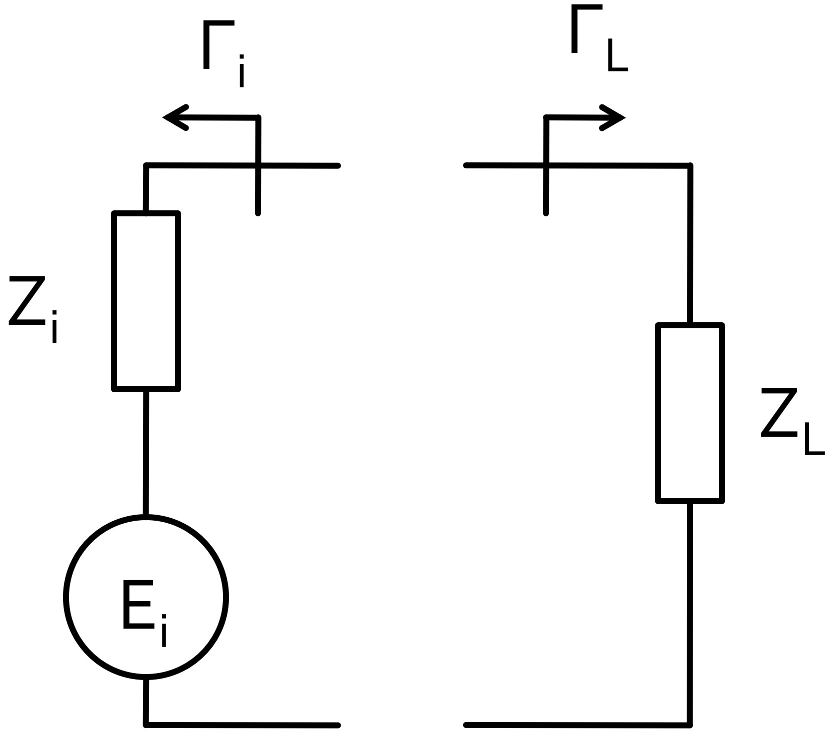
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

Matching , from the point of view of power transmission



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

Matching , from the point of view of power transmission

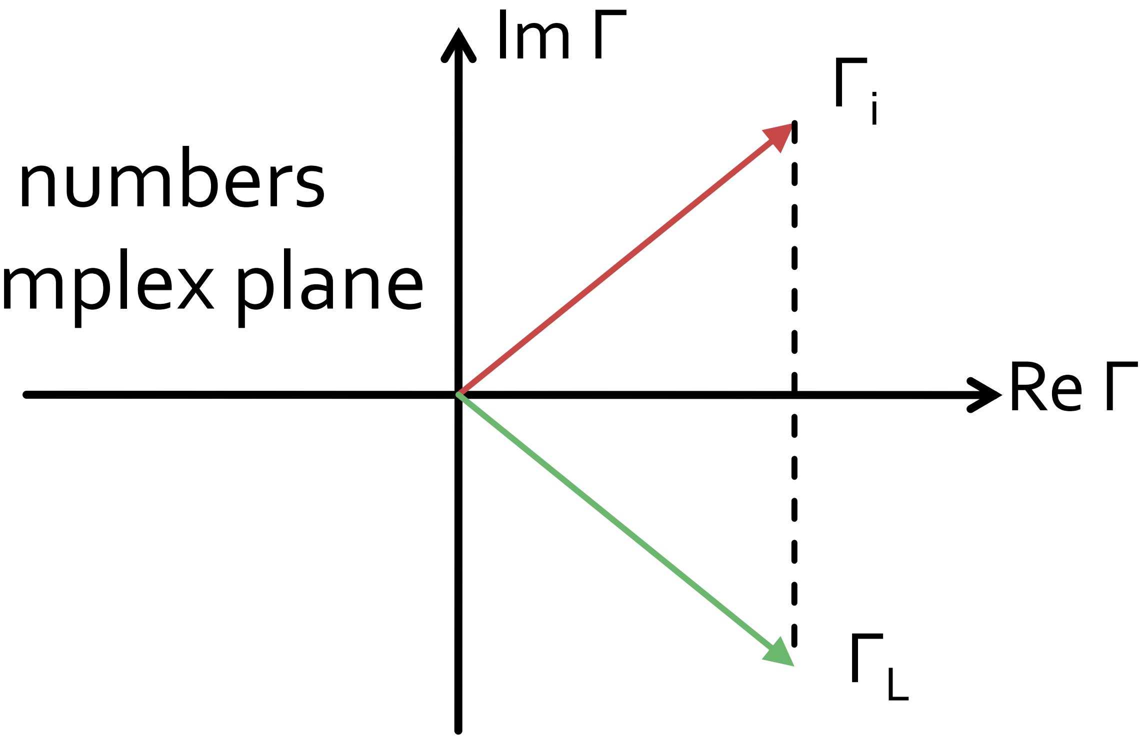
If we choose a (any) real Zo

$$Z_L = Z_i^*$$

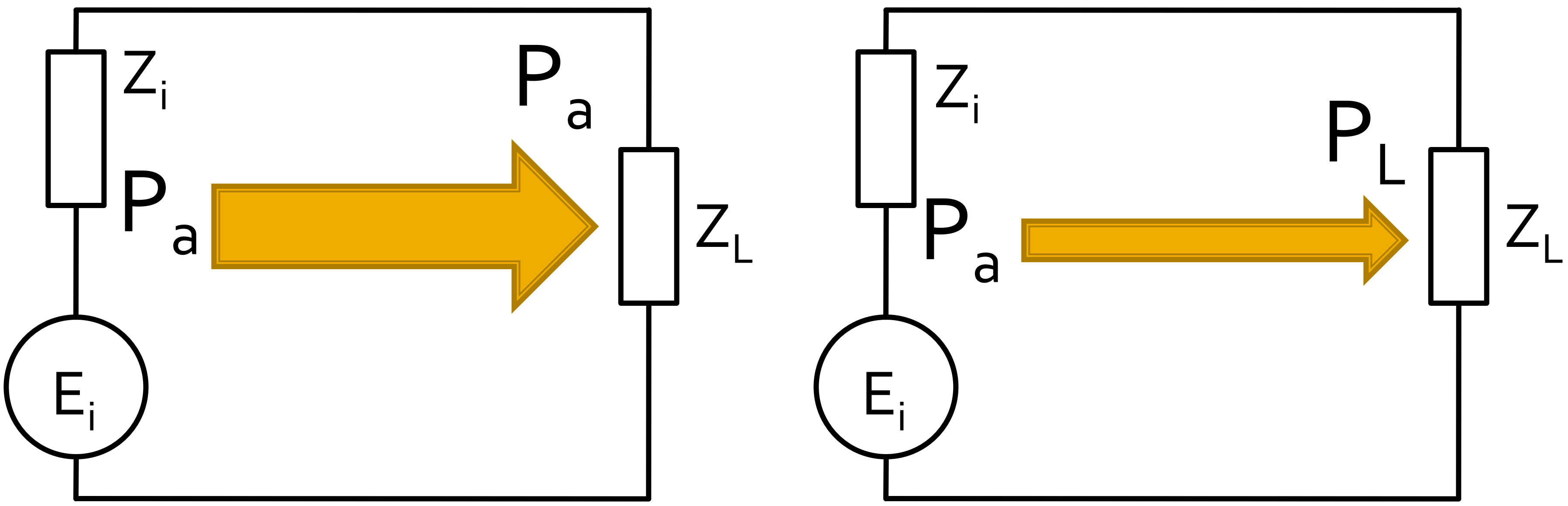
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



Reflection and power / Model



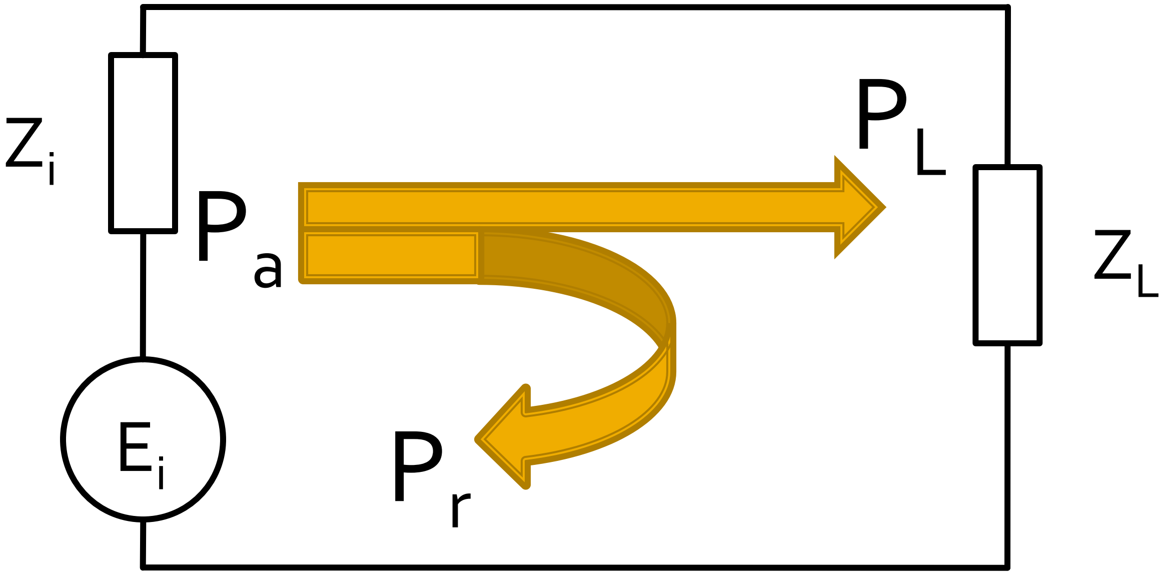
$$Z_L = Z_i^*$$

$$P_L = P_a$$

$$Z_L \neq Z_i^*$$

$$P_L < P_a$$

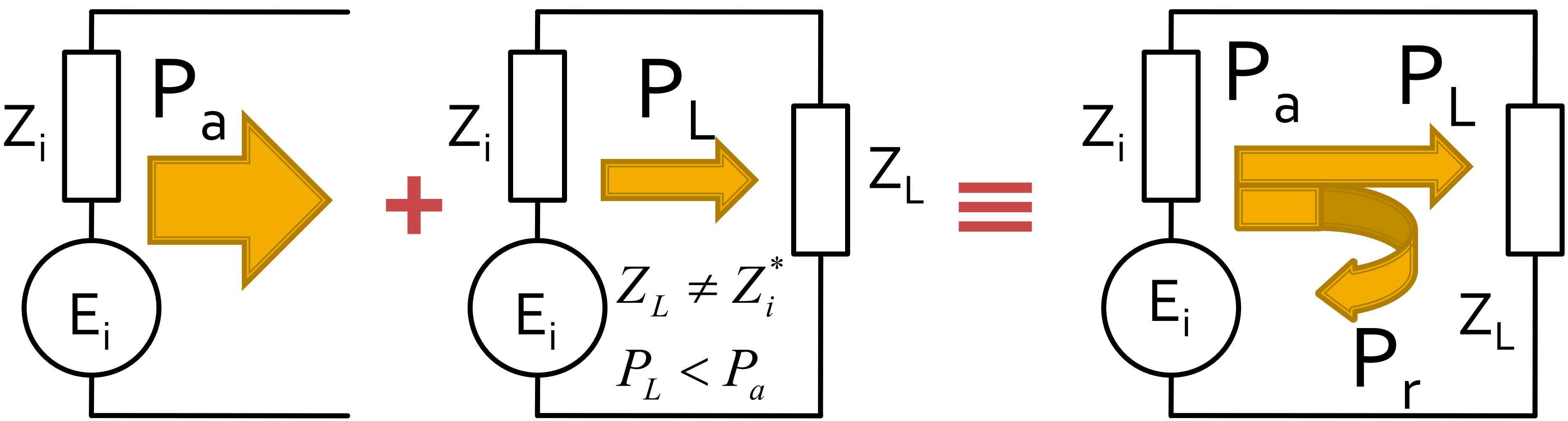
Reflection and power / Model



$$Z_L \neq Z_i^*$$

- ~~Power reflection~~
- Power of the reflected wave

Reflection and power / Model



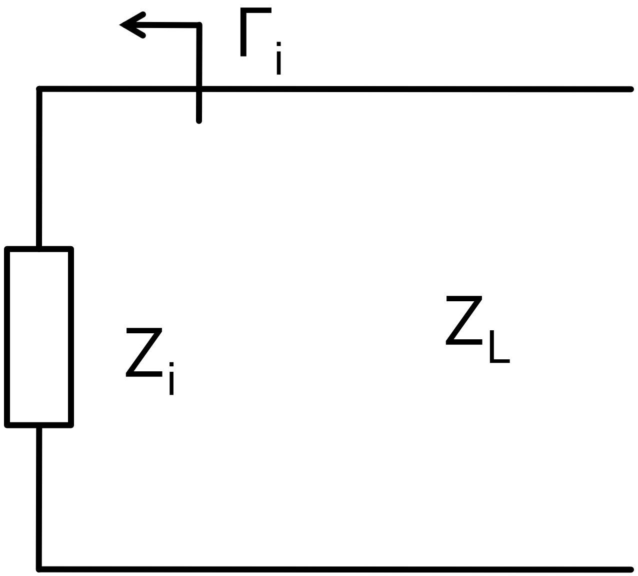
- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

The lossless line

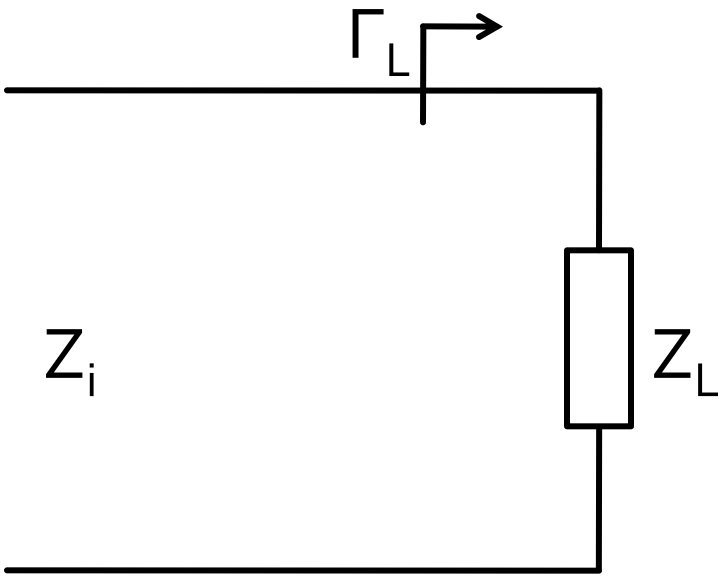
$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Average power flow is constant along the line
  - (**no** P_{avg}(**z**))
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very "energetic" (basic physics) point of view
  - more power = "more" signal

Reflection coefficient



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

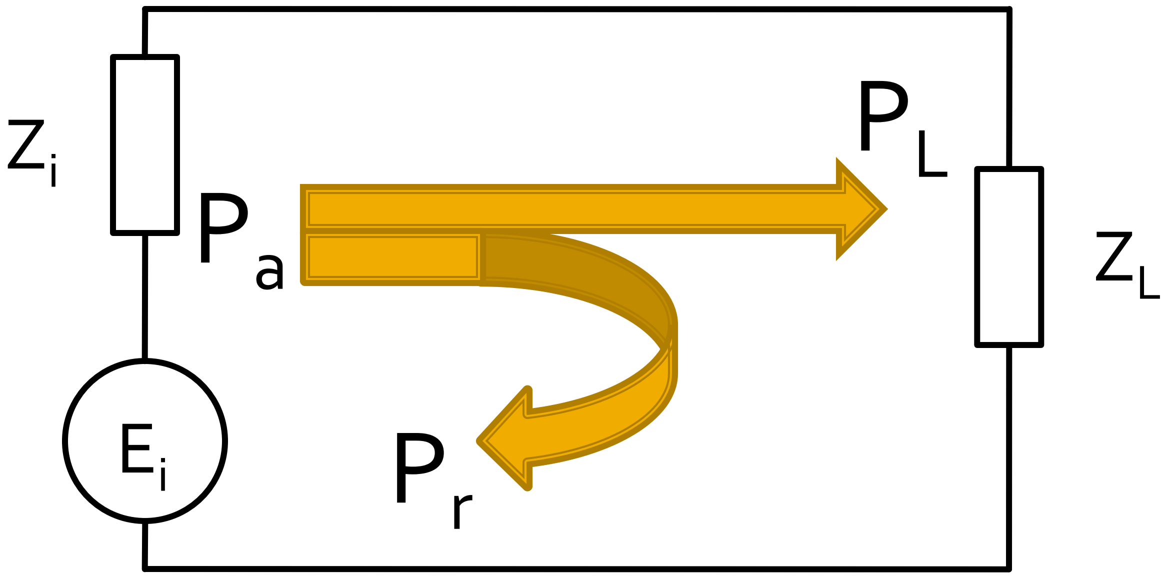
Reflection coefficient

$$\Gamma_i = \frac{(R_i - R_L) + j\cdot(X_i + X_L)}{(R_i + R_L) + j\cdot(X_i + X_L)} \qquad \Gamma_L = \frac{(R_L - R_i) + j\cdot(X_L + X_i)}{(R_L + R_i) + j\cdot(X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j\cdot(X_i + X_L)|}{|(R_i + R_L) + j\cdot(X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

Reflection and power / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[1 - \frac{4R_L \cdot R_i}{(R_i+R_L)^2 + (X_i+X_L)^2}\right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i-R_L)^2 + (X_i+X_L)^2}{(R_i+R_L)^2 + (X_i+X_L)^2}\right] = P_a \cdot |\Gamma|^2$$

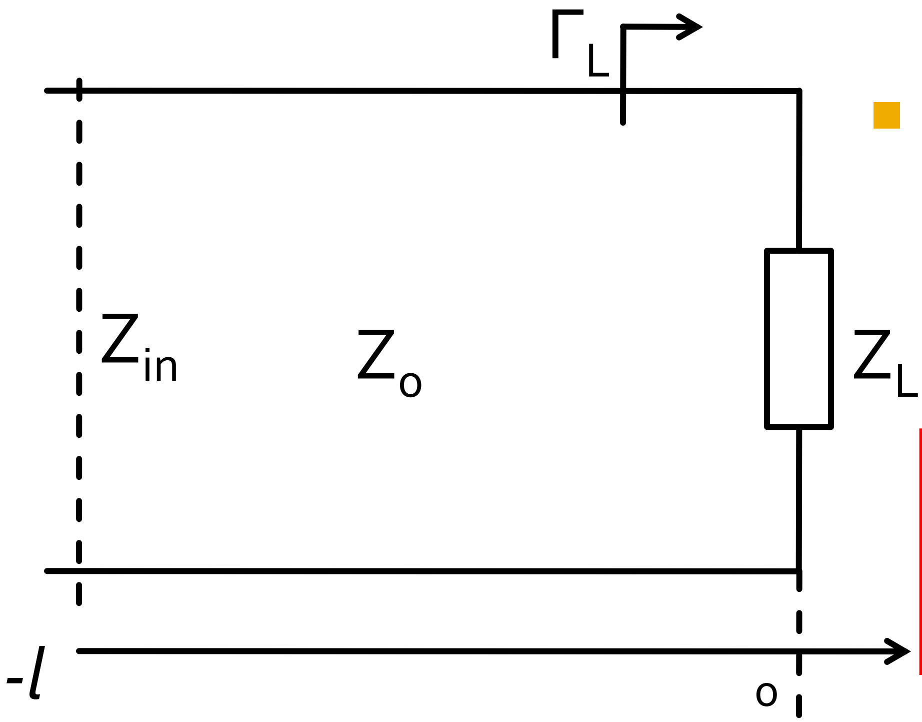
- $|\Gamma|^2$ **is** a power reflection coefficient

The quarter-wave transformer

Impedance Matching

The lossless line, special cases

- l = k·λ/2 $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$ $\tan \beta \cdot l = 0$ $Z_{in} = Z_L$
- l = λ/4 + k·λ/2 $\beta \cdot l = \frac{\pi}{2} + k \cdot \pi$ $\tan \beta \cdot l \to \infty$ $Z_{in} = \frac{Z_0^2}{Z_L}$



- quarter-wave transformer

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

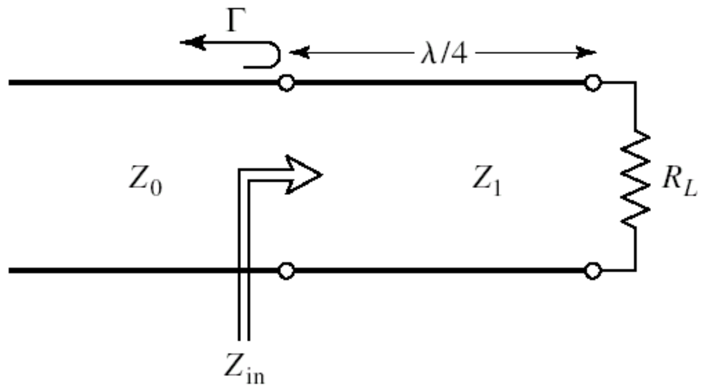
The quarter-wave transformer

- Feed line – input line with characteristic impedance Z_o
- **Real** load impedance R_L
- We desire matching the load to the feed line with a second line with the length λ/4 and characteristic impedance Z_1

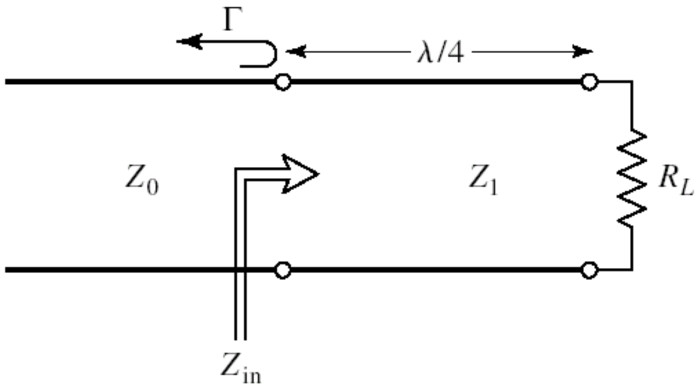
$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$



The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line (Z_o) we have only progressive wave
- In the quarter-wave line (Z_1) we have standing waves

The quarter-wave transformer

- The Multiple-Reflection Viewpoint

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \cdots$$

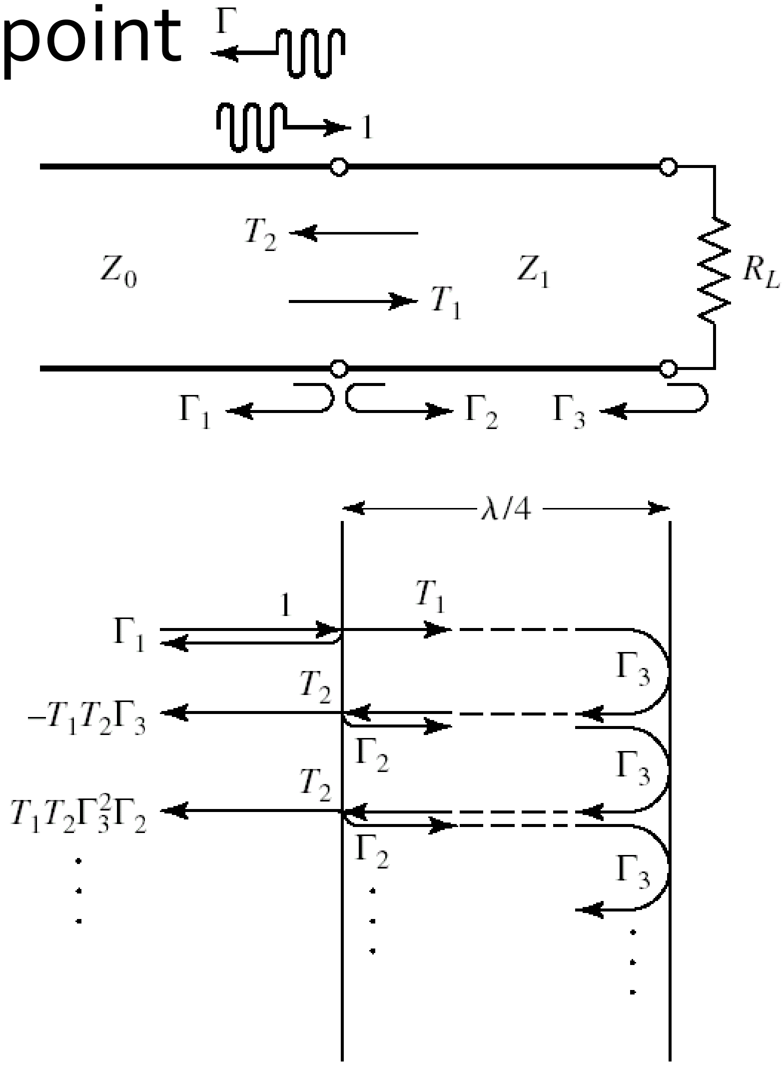
$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0}, \qquad T_2 = \frac{2Z_0}{Z_1 + Z_0}. \quad \Bigg\} \; T = 1 - \Gamma$$



The quarter-wave transformer

- The Multiple-Reflection Viewpoint

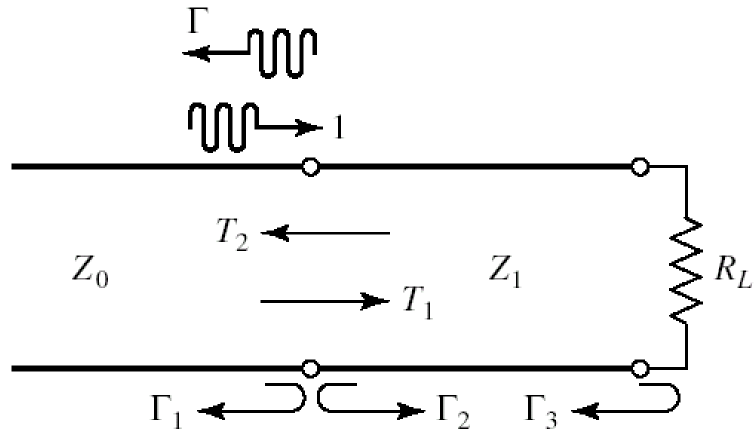
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad \text{for } |x| < 1,$$

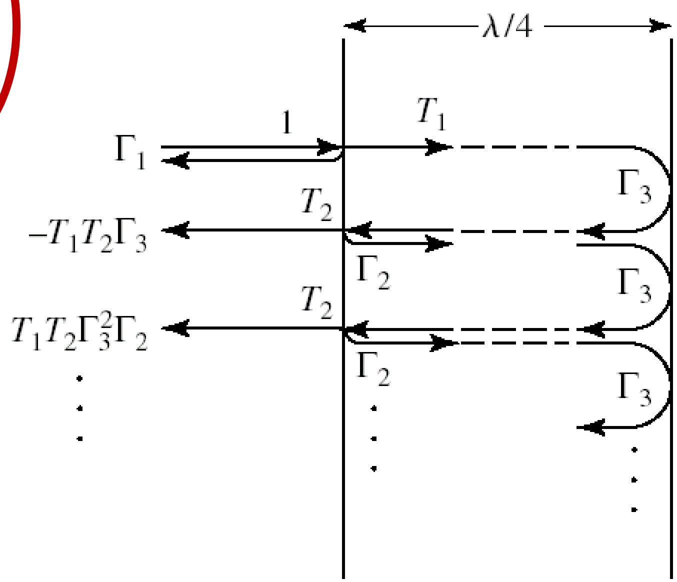
$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1 T_2\right) = \frac{2\left(Z_1^2 - Z_0 R_L\right)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma_1^2 + T_1 T_2 = \frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4 Z_1 Z_0}{(Z_1 + Z_0)^2} = 1$$

$$\Gamma = 0 \leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$





Frequency response

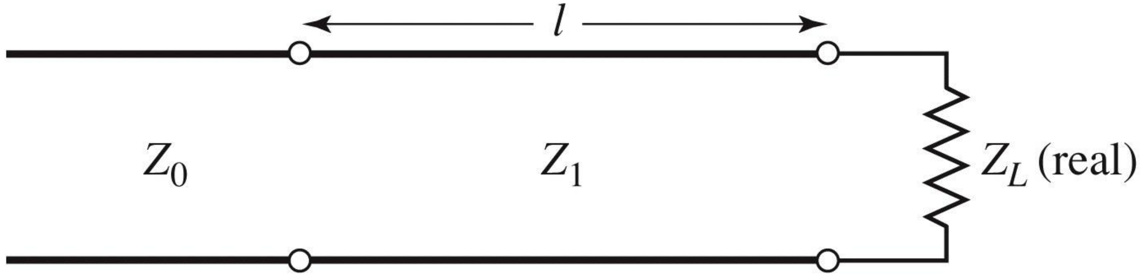


Figure 5.10

$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (**only**) at f_o

$$l = \frac{\lambda_0}{4} \qquad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta \overset{not}{=} \beta \cdot l \qquad t \overset{not}{=} \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\left(Z_1^2 - Z_0 Z_L\right)}{Z_1(Z_L + Z_0) + jt\left(Z_1^2 + Z_0 Z_L\right)}.$$

$$Z_1^2 = Z_0 \cdot Z_L$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}.$$

Frequency response

- matching quality ≡ power reflection coefficient

$$
\begin{aligned}
|\Gamma| &= \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}} \\
&= \frac{1}{\left\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2]\ \sec^2\theta\right\}^{1/2}},
\end{aligned}
$$

$$\sec\theta = \frac{1}{\cos\theta} \rightarrow$$

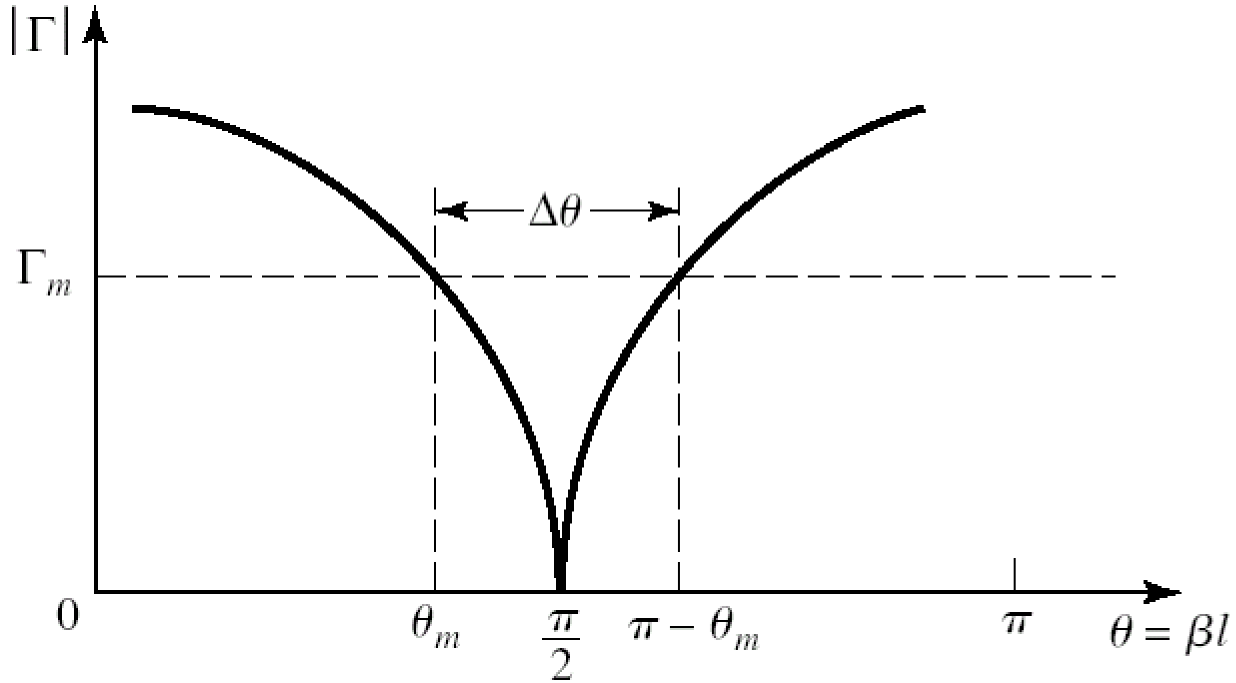
$$\sec^2\theta = 1 + \tan^2\theta = 1 + t^2$$

Frequency response

- we assume that the operating frequency is near the design frequency (narrow bandwidth)

$$f \approx f_0 \qquad l \approx \frac{\lambda_0}{4} \qquad \theta \approx \frac{\pi}{2} \qquad \sec^2\theta = 1 + \tan^2\theta >> 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos\theta|$$



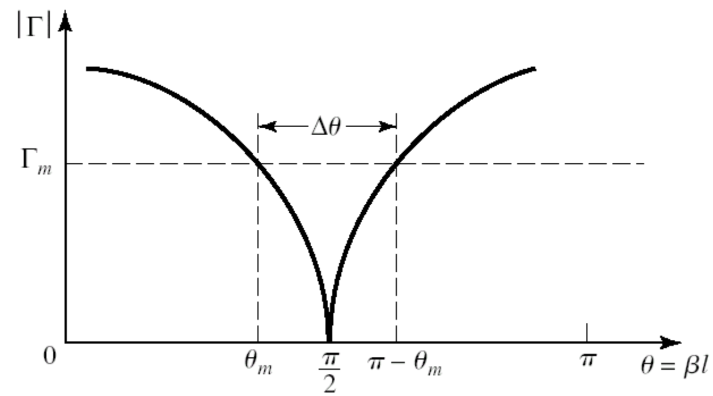
Frequency response

- we set a maximum value Γ_m for an acceptable reflection coefficient magnitude then the bandwidth of the matching transformer, θ_m

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0}\sec\theta_m\right)^2,$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$



- for TEM lines

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2 f_0}$$

$$f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|}\right]$$

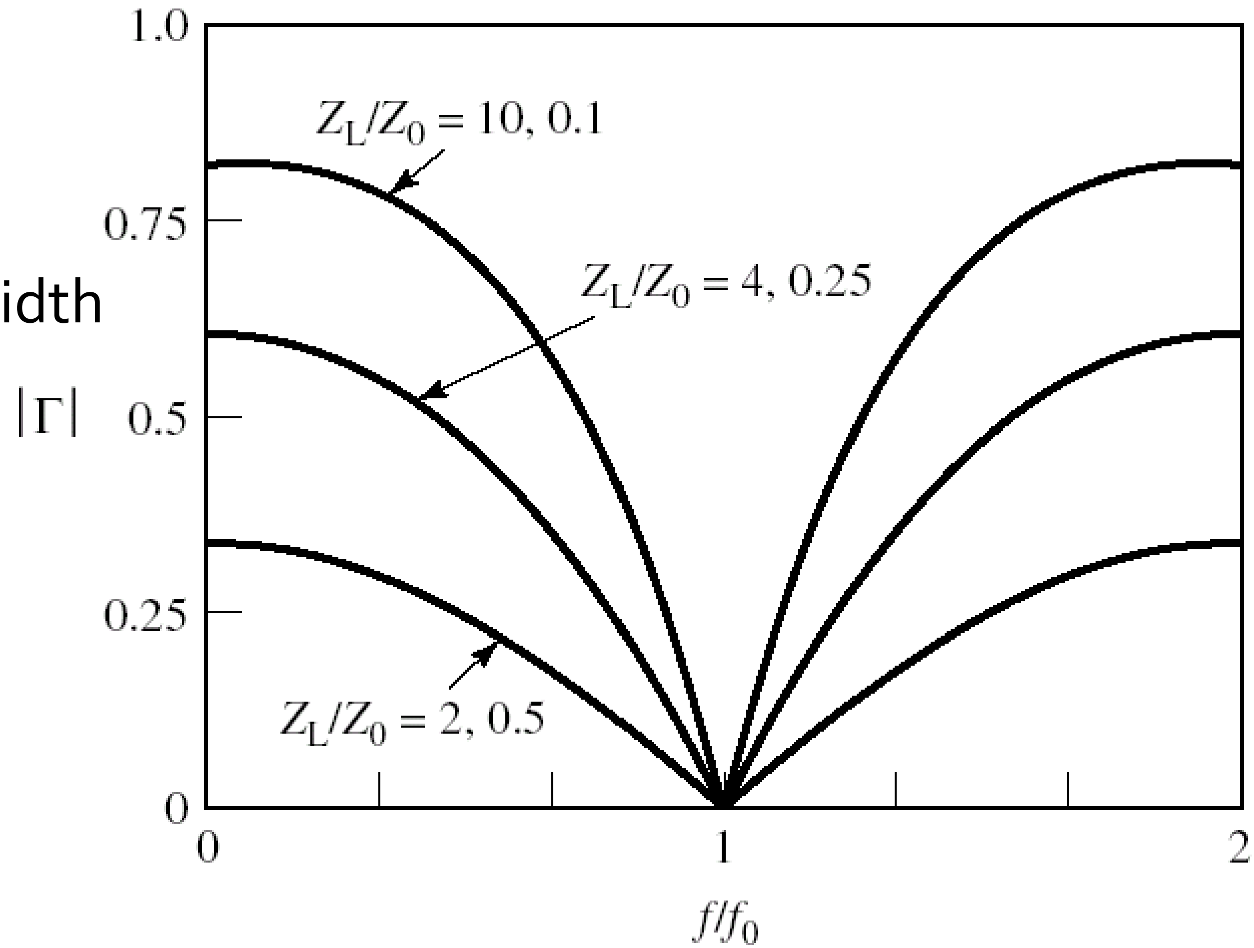
Frequency response

- When non-TEM lines (such as waveguides) are used, the propagation constant is no longer a linear function of frequency, and the wave impedance will be frequency dependent, but in practice the bandwidth of the transformer is often small enough that these complications do not substantially affect the result
- We ignored also the effect of reactances associated with discontinuities when there is a step change in the dimensions of a transmission line ($Z_0 \rightarrow Z_1$). This can often be compensated by making a small adjustment in the length of the matching section

Frequency response

- Bandwidth depends on the initial mismatch

increased bandwidth for smaller load mismatches



Example

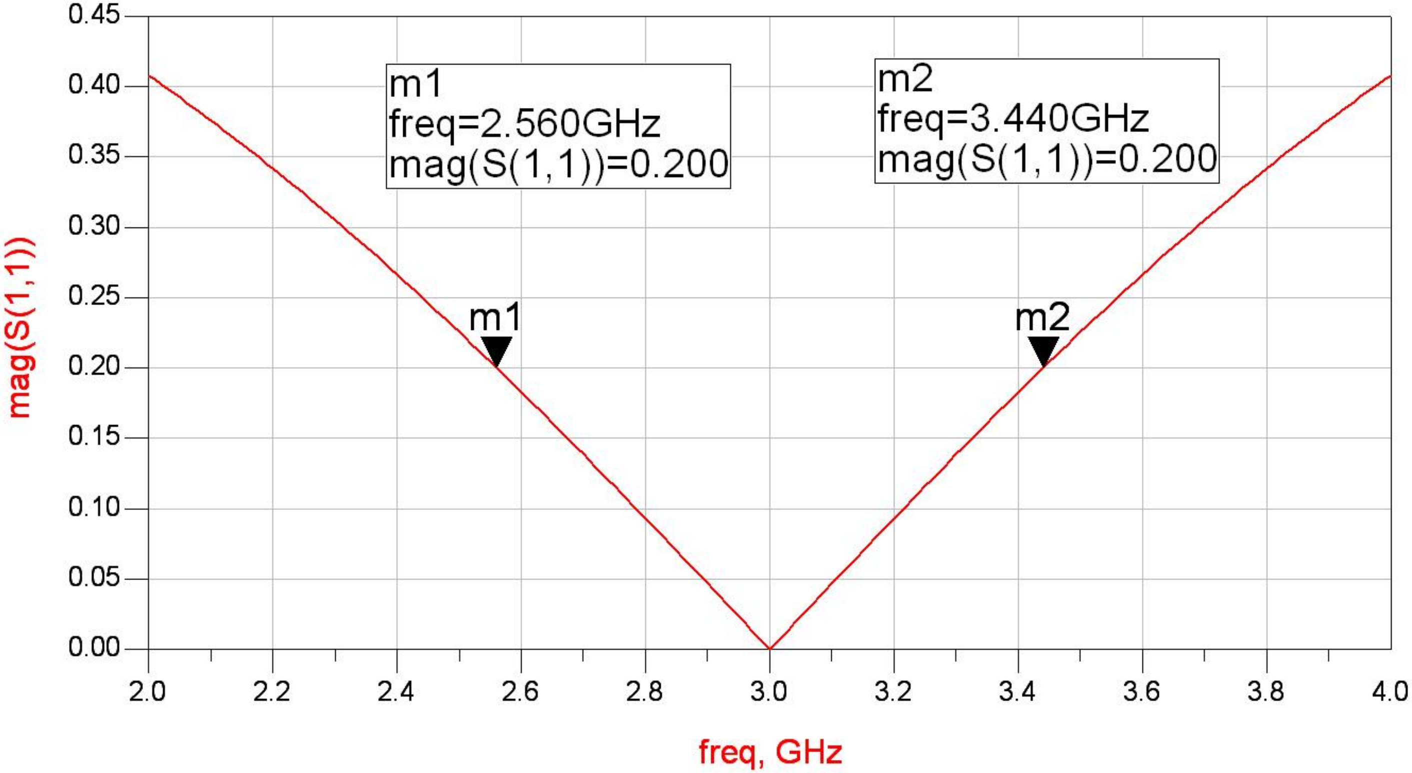
- A quarter-wave matching transformer to match a 10Ω load to a 50 Ω transmission line at f_o=3GHz
  - Determine the percent bandwidth for SWR<1.5

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega, \qquad \Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned}\frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] \\ &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0.2}{\sqrt{1-(0.2)^2}}\frac{2\sqrt{(50)(10)}}{|10-50|}\right] \\ &= 0.29, \text{ or } 29\%.\end{aligned}$$

Simulation

- ADS Simulation

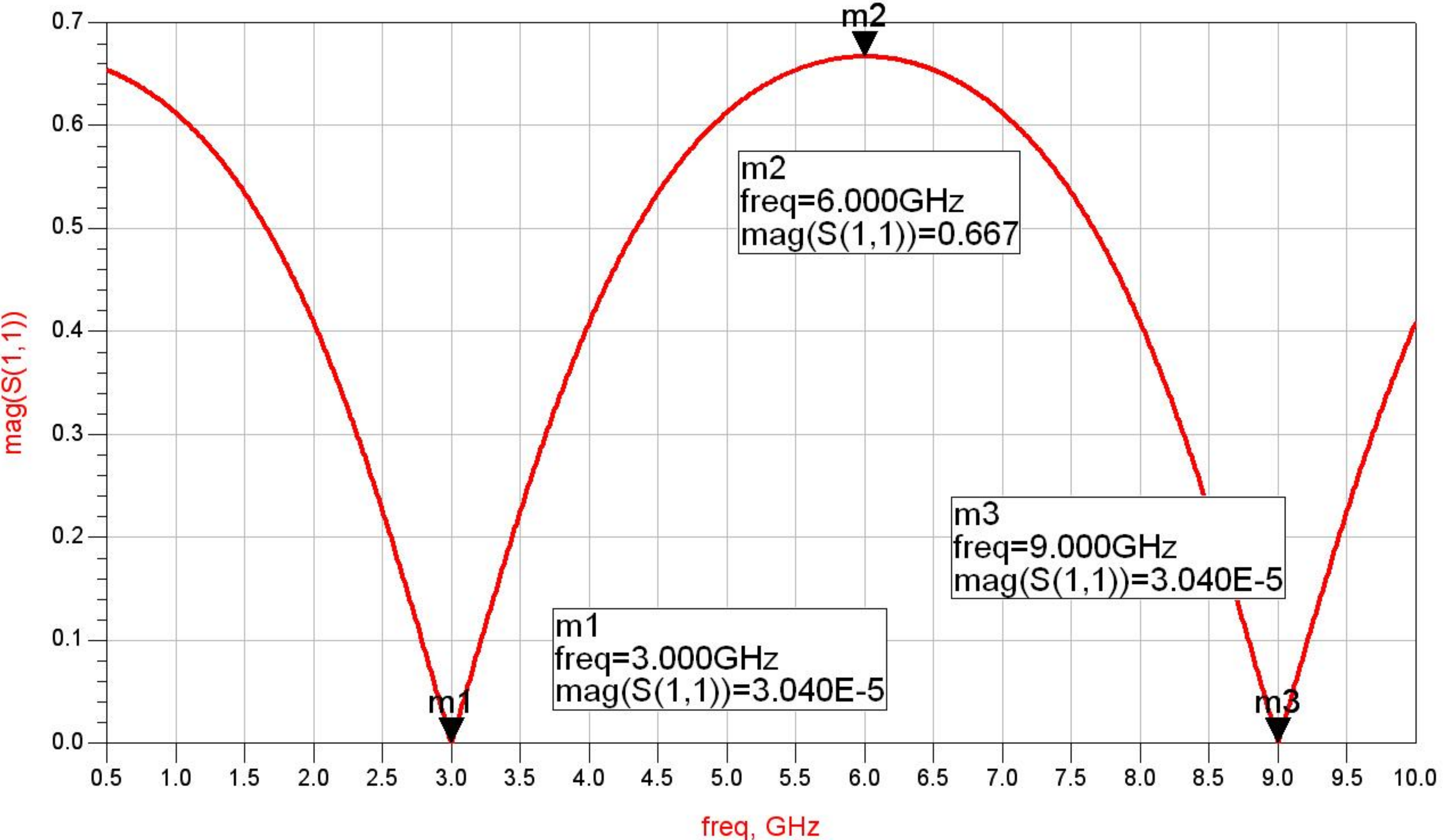


$$\Delta f = 0.88GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

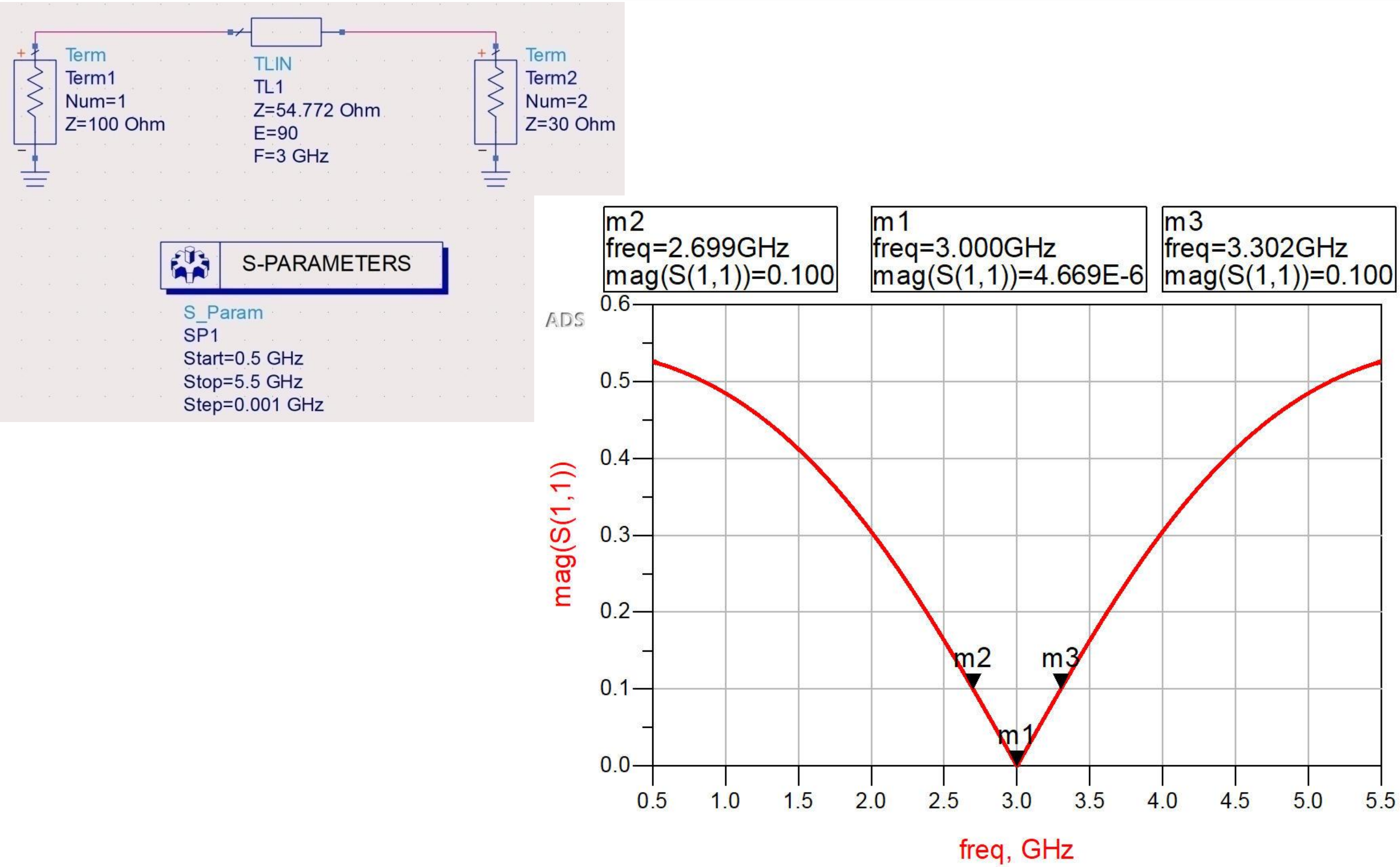
Full bandwidth simulation



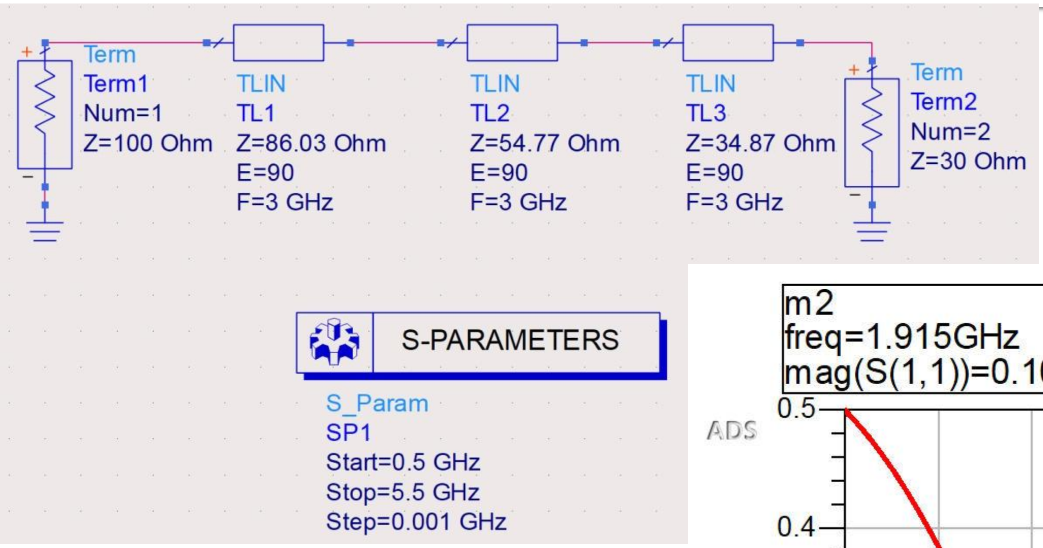
Laboratory 1

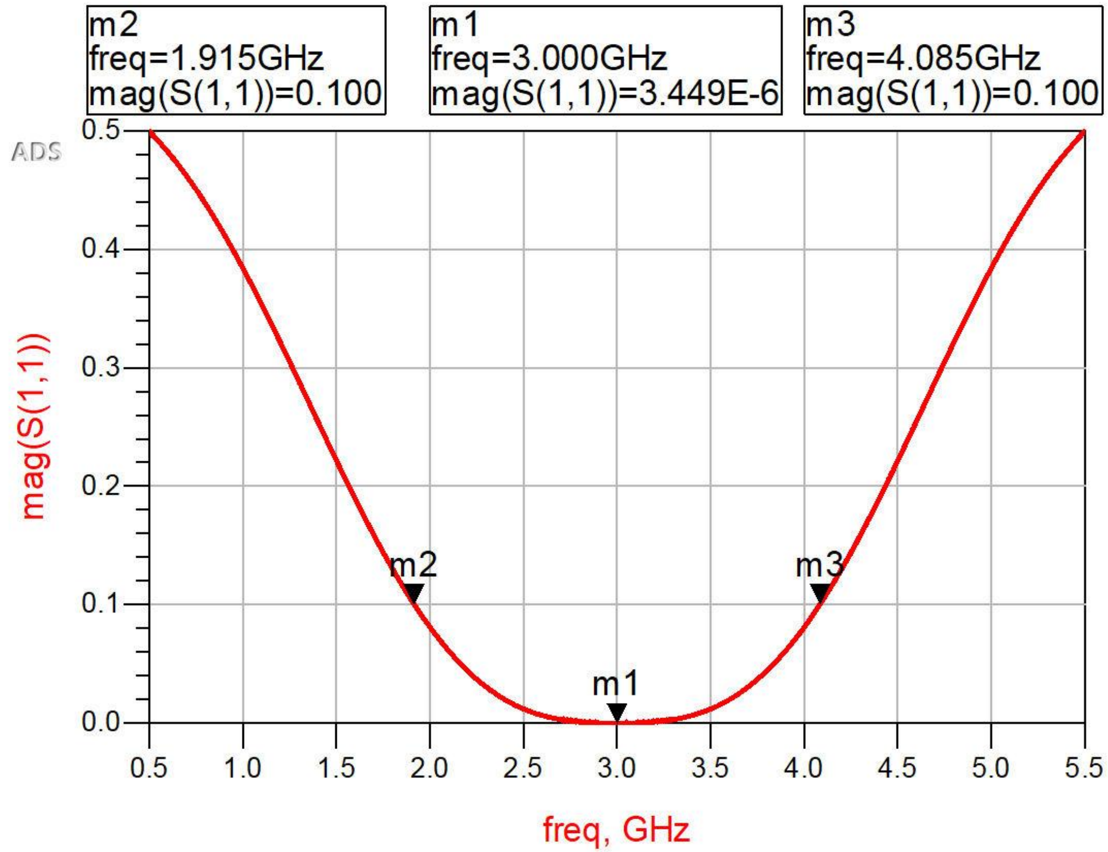
Impedance Matching

The quarter-wave transformer

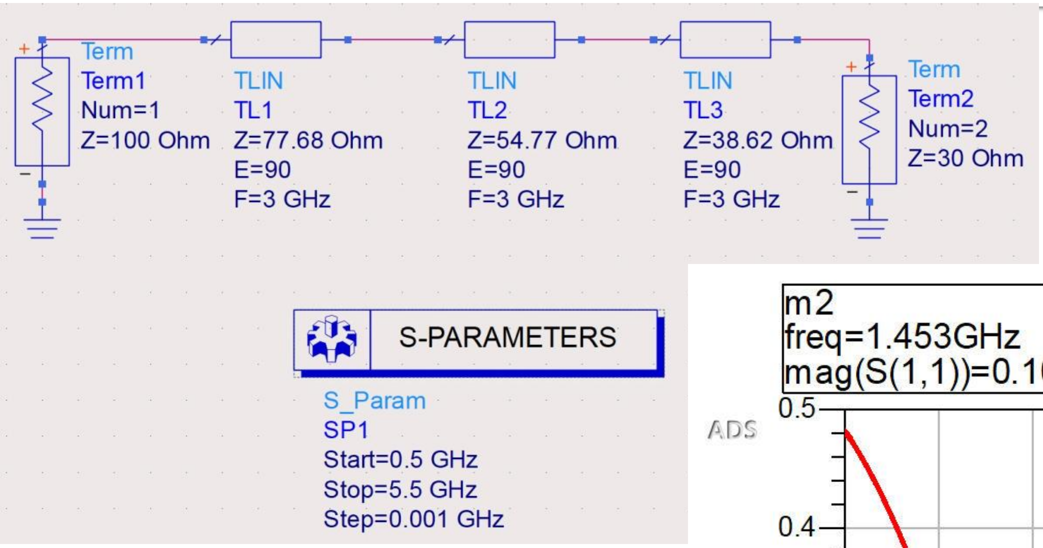


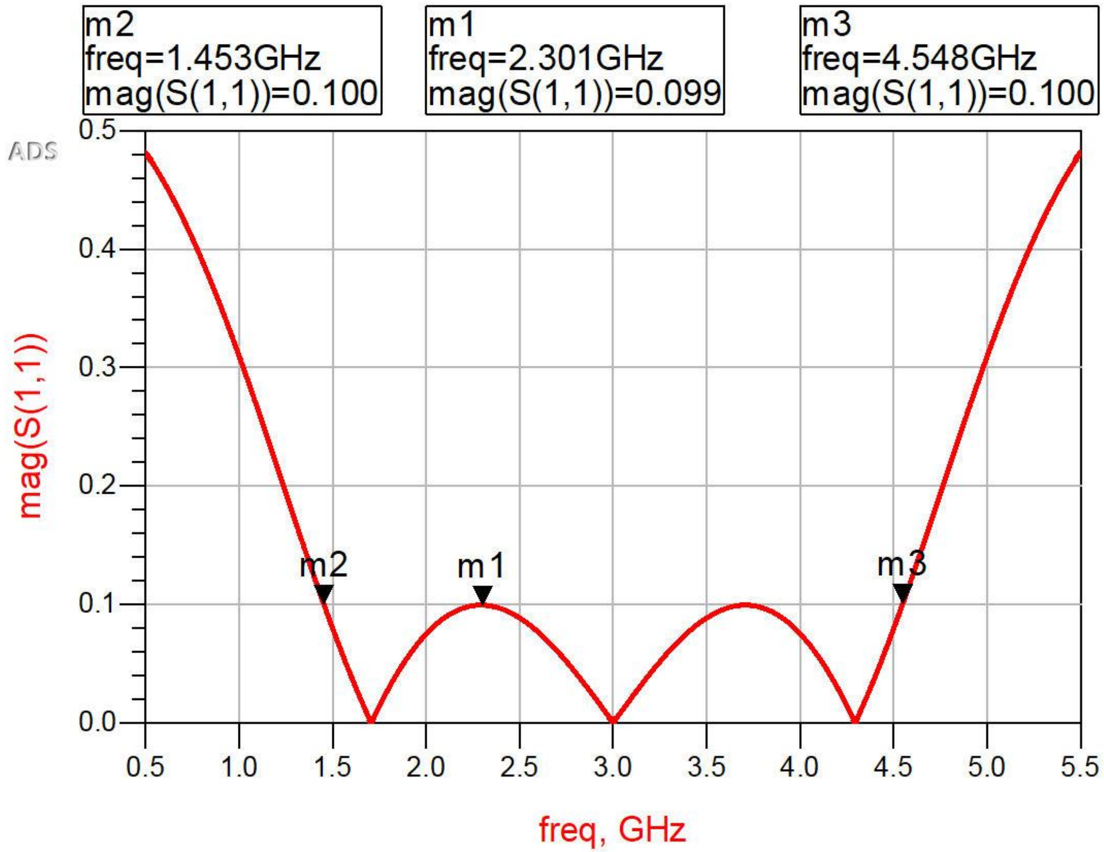
Binomial multisection transformer





Chebyshev multisection transformer





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